Perturbations of Maximal Monotone Random Operators

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ABSTRACT

Let X be a Banach space, X^* its dual, and Ω a measurable space. We study the solvability of nonlinear random equations involving operators of the form $L+T$, where L is a maximal monotone random operator from $\Omega \times X$ into X^* and $T: \Omega \times X \to X^*$ a random operator of monotone type.

1. INTRODUCTION

Let X be a Banach space, X^* its dual, and Ω a measurable space. Let T be a random operator from $\Omega \times X$ into X^*, and η a measurable mapping from Ω into X^*. The random equation corresponding to the pair $[T, \eta]$ asks for a measurable mapping $\xi: \Omega \to X$ such that for all $\omega \in \Omega$

$$T(\omega)\xi(\omega) = \eta(\omega).$$

Nonlinear random equations with operators of monotone type have been studied recently by Kannan and Salehi [10], Itoh [9], and Kravvaritis [11, 12].

It is the purpose of this paper to treat nonlinear random equations that contain operators of the form $L+T$, where L is a maximal monotone random operator from $\Omega \times X$ into X^*, and $T: \Omega \times X \to X^*$ a random operator of monotone type. More precisely, in Section 3 L is a multivalued maximal monotone random operator and T a pseudomonotone random operator. In Section 4 L is a linear maximal monotone random operator and T a random operator of type (M). Our results extend to the random case

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corresponding deterministic theorems proved by Browder [4] and Gupta [6]. In order to obtain random solutions the theorem of Kuratowski and Ryll-Nardzewski concerning the existence of measurable selections for multivalued measurable mappings is effectively used.

2. PRELIMINARIES

Let X be a real reflexive Banach space, X^* its dual, and (x^*, x) the pairing between $x^* \in X^*$ and $x \in X$. Throughout this paper Ω will denote a measurable space with a σ-algebra $\mathscr{A}$. Ω is called *complete* if there exists a complete σ-finite measure defined on $\mathscr{A}$. A mapping $F: \Omega \to 2^X$ is said to be *measurable* (*weakly measurable*) if for each closed (weakly closed) subset G of X the set $F^{-1}(G) = \{\omega \in \Omega : F(\omega) \cap G \neq \varnothing\}$ belongs to $\mathscr{A}$. We denote by $B(\Omega, X)$ the set of all measurable mappings $\xi: \Omega \to X$ such that $\sup\{\|\xi(\omega)\|, \omega \in \Omega\} < \infty$. The symbols $\to$ and $\rightharpoonup$ are used to denote strong and weak convergence, respectively. Let T be an operator from $D \subset X$ into 2^{X^*}. T is said to be: (1) *monotone* if $(x^* - y^*, x - y) \geqslant 0$ for all $x, y \in D$ and $x^* \in T(x)$ and $y^* \in T(y)$, and (2) *maximal monotone* if it is monotone and its graph is not properly contained in the graph of any other monotone operator T_1 from X into 2^{X^*}. If T is single-valued, then T is said to be: (1) *demicontinuous* if for any sequence $\{x_n\}$ in D with $x_n \to x \in D$, it follows that $Tx_n \rightharpoonup Tx$, and (2) *bounded* if for each bounded subset B of D, $T(B)$ is a bounded subset of X^*.

Let D be a subset of X, and T an operator from $\Omega \times D$ into 2^{X^*}. T is called *random* if for any $x \in D$, $T(\cdot)x$ is measurable. A random operator T is called *coercive* if there exists a function $c: \mathbb{R}^+ \to \mathbb{R}$ with $\lim_{r \to \infty} c(r) = +\infty$ such that $(x^*, x) \geqslant c(\|x\|)\|x\|$ for all $\omega \in \Omega$, $x \in D$, and $x^* \in T(\omega)x$. A random operator T is said to be monotone (demicontinuous, etc.) if for each $\omega \in \Omega$, $T(\omega)$ is monotone (demicontinuous, etc.)

3. PERTURBATIONS OF NONLINEAR MONOTONE RANDOM OPERATORS

Let Y and Z be topological spaces. We recall (see e.g. [16]) that a mapping $T: Y \to 2^Z$ is said to be *lower semicontinuous* if the set $\{y \in Y : T(y) \cap G \neq \varnothing\}$ is open for each open subset G of Z.

Let X be a separable reflexive Banach space. We note that the dual space X^* endowed with the weak topology satisfies the first axiom of countability [8, p. 64]. As in the proof of [16, Lemma 4], one can show that a mapping

$T: X \to 2^{X^*}$ (X^* taken with its weak topology) is lower semicontinuous if and only if the relations $x_n \to x$ and $y \in Tx$ imply the existence of a sequence $\{y_n\}$ with $y_n \in Tx_n$ such that $y_n \rightharpoonup y$.

DEFINITION. Let K be a closed convex subset of the reflexive Banach space X, and T an operator from K into X^*. T is called *pseudomonotone* if the following conditions hold:

(i) For each finite-dimensional subspace F of X, the operator T is demicontinuous from $K \cap F$ into X^*.

(ii) For any sequence $\{x_n\}$ in K such that $x_n \rightharpoonup x$, $Tx_n \rightharpoonup x^*$, and lim sup $(Tx_n, x_n - x) \leqslant 0$, we have $x^* = Tx$ and $\lim\,(Tx_n, x_n) = (x^*, x)$.

The concept of pseudomonotone operators was first introduced by Brézis in [1] using filters. For the above definition of pseudomonotonicity and related results we refer to [3]. We note that any demicontinuous monotone operator from X into X^* is pseudomonotone.

We shall need the following lemma.

LEMMA. *Let Ω be complete, X a separable reflexive Banach space, and D a subset of X with $0 \in D$. Let $L: \Omega \times D \to 2^{X^*}$ be a monotone random operator such that L is lower semicontinuous (X^* taken with its weak topology) and $L(\omega)x$ is a closed subset of X^* for each $\omega \in \Omega$ and $x \in D$. Suppose further that for each $\omega \in \Omega$ there exists $u(\omega) \in L(\omega)0$ such that $\sup\{\|u(\omega)\|: \omega \in \Omega\} = M_1 < \infty$. Let F be a finite-dimensional subspace of X, and $T: \Omega \times F \to X^*$ be a pseudomonotone, bounded, coercive, and random operator. Then there exists $\xi \in B(\Omega, F)$ such that*

$$(v + T(\omega)\xi(\omega), y - \xi(\omega)) \geqslant 0 \qquad \text{for all} \quad \omega \in \Omega, \quad y \in F \cap D \quad \text{and} \quad v \in L(\omega)y.$$

Proof. By [15, p. 118], for each $\omega \in \Omega$, there exists $x \in F$ such that

$$(v + T(\omega)x, y - x) \geqslant 0 \qquad \text{for all} \quad y \in D \cap F \text{ and } v \in L(\omega)y.$$

Setting $y = 0$ and $v = u(\omega)$ in this inequality, we obtain

$$(T(\omega)x, x) \leqslant (u(\omega), -x) \leqslant \|u(\omega)\|\|x\| \leqslant M_1\|x\|.$$

Since T is coercive, we get $c(\|x\|)\|x\| \leqslant M_1\|x\|$. It follows from the growth property of $c(r)$ that there exists $M > 0$ such that $\|x\| \leqslant M$. Let $B = \{x \in F : \|x\| \leqslant M\}$. Define a mapping $G: \Omega \to 2^B$ by

$$G(\omega) = \{x \in B : (v + T(\omega)x, y - x) \geqslant 0 \quad \text{for all} \quad y \in F \cap D \text{ and } v \in L(\omega)y\}.$$

Let $\{y_n\}$ be a sequence of points in $F \cap D$ whose union is dense in $F \cap D$. By the lower semicontinuity of L we have

$$G(\omega) = \bigcap_{n=1}^{\infty} \{x \in B : (v + T(\omega)x, y_n - x) \geqslant 0 \quad \text{for all} \quad v \in L(\omega)y_n\}.$$

By [7, Theorem 4.1], in order to prove that G is measurable, it suffices to show that for a given $y \in F \cap D$ the mapping

$$\Gamma(\omega) = \{x \in B : (v + T(\omega)x, y - x) \geqslant 0 \quad \text{for all} \quad v \in L(\omega)y\}$$

is measurable. By [7], there exists a sequence $\{v_n(\omega)y\}$ of measurable selectors for L such that $\overline{\{v_n(\omega)y\}} = L(\omega)y$ for all $\omega \in \Omega$. Now, we have

$$\Gamma(\omega) = \bigcap_{n=1}^{\infty} \{x \in B : (v_n(\omega)y + T(\omega)x, y - x) \geqslant 0\}.$$

For each $n \in \mathbb{N}$, the mapping $f_n : \Omega \times B \to \mathbb{R}$ defined by $f_n(\omega, x) = (v_n(\omega)y + T(\omega)x, y - x)$ is measurable with respect to ω and continuous with respect to x. Thus the mapping

$$\Gamma_n(\omega) = \{x \in B : (v_n(\omega)y + T(\omega)x, y - x) \geqslant 0\}$$

is measurable [7]. Therefore, the same holds for Γ. Then G is measurable, and by [13] it admits a measurable selection ξ, i.e., there exists a measurable mapping $\xi : \Omega \to B$ such that

$$(v + T(\omega)\xi(\omega), y - \xi(\omega)) \geqslant 0 \quad \text{for all} \quad \omega \in \Omega, \quad y \in F \cap D, \text{ and} \quad v \in L(\omega)y. \qquad \blacksquare$$

The deterministic case corresponding to the following theorem was obtained by Pascali and Sburlan [15, p. 120] and Browder [4, Theorem 7.8].

THEOREM 1. *Let Ω be complete, X a separable reflexive Banach space, and D a subset of X with $0 \in D$. Let $L: \Omega \times D \to 2^{X^*}$ be a maximal monotone random operator such that L is lower semicontinuous (X^* taken with its weak topology). Suppose that for each $\omega \in \Omega$ there exists $u(\omega) \in L(\omega)0$ such that $\sup\{\|u(\omega)\|: \omega \in \Omega\} = M_1 < \infty$. Let $T: \Omega \times X \to X^*$ be a pseudomonotone, bounded, coercive random operator. Then for each $\eta \in B(\Omega, X^*)$ there exists $\xi \in B(\Omega, X)$ such that*

$$\eta(\omega) \in L(\omega)\xi(\omega) + T(\omega)\xi(\omega) \qquad \text{for all} \quad \omega \in \Omega.$$

Proof. We may assume that $\eta(\omega) = 0$ for all $\omega \in \Omega$. Let $\{X_n\}$ be an increasing sequence of finite-dimensional subspaces of X such that $\bigcup_n X_n$ is dense in X and $\bigcup_n (D \cap X_n)$ is dense in D. By Lemma, for each n, there exists $\xi_n \in B(\Omega, X_n)$ such that

$$(v + T(\omega)\xi_n(\omega), y - \xi_n(\omega)) \geqslant 0 \tag{1}$$

for all $\omega \in \Omega$, $y \in D \cap X_n$, and $v \in L(\omega)y$. Setting $y = 0$ and $v = u(\omega)$ in this inequality, we get

$$(T(\omega)\xi_n(\omega), \xi_n(\omega)) \leqslant (u(\omega), -\xi_n(\omega)) \leqslant M_1 \|\xi_n(\omega)\|.$$

By the coercivity of T we conclude that there exists $M > 0$ such that $\|\xi_n(\omega)\| \leqslant M$ for all $\omega \in \Omega$ and $n \in \mathbb{N}$. Since the ball $B_M = \{x \in X: \|x\| \leqslant M\}$ is a metrizable separable space in the weak topology, the mappings $\Gamma_n(\omega) = \text{weakcl}\{\xi_i(\omega): i \geqslant n\}$ are weakly measurable [7, p. 62]. Then the mapping $\Gamma(\omega) = \bigcap_n \Gamma_n(\omega)$ is also weakly measurable [7]. By [13], Γ admits a weakly measurable selection $\xi: \Omega \to B_M$ that is also measurable [5, p. 149]. For a fixed $\omega \in \Omega$, there is a subsequence $\{\xi_k(\omega)\}$ of $\{\xi_n(\omega)\}$ such that $\xi_k(\omega) \rightharpoonup \xi(\omega)$. Since T is bounded, we may assume that $T(\omega)\xi_k(\omega) \rightharpoonup x^*(\omega)$. From (1) we have

$$(T(\omega)\xi_k(\omega), \xi_k(\omega)) \leqslant (T(\omega)\xi_k(\omega), y) + (v, y - \xi_k(\omega)) \tag{2}$$

for all $y \in D \cap X_k$ and $v \in L(\omega)y$. We assert that

$$\limsup (T(\omega)\xi_k(\omega), \xi_k(\omega)) \leqslant (x^*(\omega), y) + (v, y - \xi(\omega)) \tag{3}$$

for all $y \in \bigcup_k (D \cap X_k)$ and $v \in L(\omega)y$. Indeed, let y be any element of $\bigcup_k (D \cap X_k)$. Then y lies in $D \cap X_m$ for some m, and since $\{D \cap X_k\}$

increases with k, $y \in D \cap X_k$ for all $k \geqslant m$. Thus (2) holds for all $k \geqslant m$, which in turn implies (3). Since $\bigcup_k (D \cap X_k)$ is dense in D, and L is lower semicontinuous, we conclude that (3) holds for all $y \in D$ and $v \in L(\omega)y$. The rest of the proof proceeds as in the proof of [15, p. 119]. Finally, we have $-T(\omega)\xi(\omega) \in L(\omega)\xi(\omega)$, i.e., $0 \in L(\omega)\xi(\omega) + T(\omega)\xi(\omega)$, which completes the proof. ■

A basic consequence of Theorem 1 is the following result, which extends to the multivalued case a result proved in [12].

THEOREM 2. *Let Ω be complete, X a separable reflexive Banach space, and D a subset of X with $0 \in D$. Let $L: \Omega \times X \to 2^{X^*}$ be a coercive, maximal monotone random operator such that L is lower semicontinuous (X^* taken with its weak topology). Suppose that for each $\omega \in \Omega$ there exists $u(\omega) \in L(\omega)0$ such that $\sup\{\|u(\omega)\| : \omega \in \Omega\} < \infty$. Then for each $\eta \in B(\Omega, X^*)$ there exists $\xi \in B(\Omega, X)$ such that*

$$\eta(\omega) \in L(\omega)\xi(\omega) \qquad \text{for all} \quad \omega \in \Omega.$$

Proof. We may assume that $\eta(\omega) = 0$ for all $\omega \in \Omega$. Let J be the duality mapping defined by $Jx = \{x^* \in X^* : (x^*, x) = \|x\|^2, \ \|x\| = \|x^*\|\}$. By a result of Trojanski [17] we may assume that X and X^* are locally uniformly convex. Thus the mapping J is single-valued, demicontinuous, and monotone. It then follows that J is pseudomonotone [3, Proposition 8]. By Theorem 1, for each $\varepsilon > 0$, there exists $\xi_\varepsilon \in B(\Omega, X)$ and $u_\varepsilon(\omega) \in L(\omega)\xi_\varepsilon(\omega)$ such that

$$0 = u_\varepsilon(\omega) + \varepsilon J\xi_\varepsilon(\omega).$$

We have

$$0 = \big(u_\varepsilon(\omega) + \varepsilon J\xi_\varepsilon(\omega), \xi_\varepsilon(\omega)\big) \geqslant c\big(\|\xi_\varepsilon(\omega)\|\big)\|\xi_\varepsilon(\omega)\| + \varepsilon\|\xi_\varepsilon(\omega)\|^2,$$

which implies that $c(\|\xi_\varepsilon(\omega)\|) \leqslant 0$. So there exists $M > 0$ such that $\|\xi_\varepsilon(\omega)\| \leqslant M$ for all $\varepsilon > 0$ and $\omega \in \Omega$. Let $\{\varepsilon_n\}$ be a sequence of positive numbers such that $\varepsilon_n \to 0$. For each n, we set $\xi_n(\omega) = \xi_{\varepsilon_n}(\omega)$ and $u_n(\omega) = u_{\varepsilon_n}(\omega)$. As in the proof of Theorem 1, there exists $\xi \in B(\Omega, X)$ such that for a fixed $\omega \in \Omega$, there is a subsequence $\{\xi_k(\omega)\}$ of $\{\xi_n(\omega)\}$ such that $\xi_k(\omega) \rightharpoonup \xi(\omega)$. Let $[x, x^*]$ be any element of $G(L(\omega))$. By the monotonicity of L we have

$$\big(u_k(\omega) - x^*, \xi_k(\omega) - x\big) \geqslant 0,$$

or

$$(-\varepsilon_k J\xi_k(\omega) - x^*, \xi_k(\omega) - x) \geqslant 0.$$

Letting $k \to \infty$, we get

$$(x^*, x - \xi(\omega)) \geqslant 0.$$

Since L is maximal monotone, we conclude that $0 \in L(\omega)\xi(\omega)$, and the proof is complete. ■

4. PERTURBATIONS OF LINEAR MONOTONE RANDOM OPERATORS

We shall need the following proposition.

PROPOSITION. *Let X be a separable Banach space, and D a dense linear subspace of X. Let $L: \Omega \times D \to X$ be a closed linear random operator such that for each $\omega \in \Omega$, $L(\omega)$ is one to one and onto. Then the operator $S: \Omega \times X \to X$ defined by $S(\omega)x = L(\omega)^{-1}x$ $(\omega \in \Omega,\ x \in X)$ is random.*

Proof. For a fixed $\omega_0 \in \Omega$, let X_1 be the subspace D endowed with the norm $\|x\|_1 = \|L(\omega_0)x\|$. This norm is equivalent to the graph norm $\|x\| + \|L(\omega_0)x\|$, and with it X_1 becomes a separable Banach space. By the closed-graph theorem, $L(\omega)L^{-1}(\omega_1)$ is a bounded linear operator for every ω, $\omega_1 \in \Omega$. Hence, for each $\omega \in \Omega$, $x \to \|L(\omega)x\|$ defines a norm on D which is equivalent to the norm $\|x\|_1$. So, for each ω, $L(\omega)$ is a bounded linear operator from X_1 onto X. By [14], for each $x \in X$, $S(\cdot)x: \Omega \to X_1$ is a random operator. Since the injection of X_1 into X is continuous, $S(\cdot)x$ is also random as an operator from Ω into X. ■

DEFINITION 2. Let X be a reflexive Banach space, and T an operator from X into X^*. T is said to be *of type* (M) if the following conditions hold:

(i) T is continuous from finite-dimensional subspaces of X to X^* endowed with weak topology.

(ii) For any sequence $\{x_n\}$ in X such that $x_n \rightharpoonup x$, $Tx_n \rightharpoonup x^*$, and $\limsup(Tx_n, x_n - x) \leqslant 0$, we have $x^* = Tx$.

The concept of operators of type (M) was introduced by Brézis in [1], using filters. For the above definition and related properties we refer to [15].

Now, we generalize Theorem 1 in [6] as follows.

THEOREM 3. *Let Ω be a measurable space, H a separable Hilbert space, and D a dense linear subspace of H. Let $L: \Omega \times D \to H$ be a linear maximal monotone random operator, and $T: \Omega \times H \to H$ a random operator that is bounded, of type (M), and coercive. Then for each $\eta \in B(\Omega, H)$ there exists $\xi \in B(\Omega, H)$ such that*

$$L(\omega)\xi(\omega) + T(\omega)\xi(\omega) = \eta(\omega) \qquad \text{for all} \quad \omega \in \Omega.$$

Proof. We may assume that $\eta(\omega) = 0$ for all $\omega \in \Omega$. For each $\omega \in \Omega$, let $L_\varepsilon(\omega)$ be the Yosida approximants of $L(\omega)$ defined by $L_\varepsilon(\omega)x = \varepsilon^{-1}[I - J_\varepsilon(\omega)]x$, where $J_\varepsilon(\omega)x = [I + \varepsilon L(\omega)]^{-1}x$ (see [2, p. 102]). By Proposition, J_ε is a random operator, and so the same holds for L_ε. Moreover, L_ε is monotone (cf. [2, Proposition VII.2]). Now, the operator $L_\varepsilon + T$ is random, bounded, coercive, and of type (M) [1, Proposition 18]. By [11], there exists $\xi_\varepsilon \in B(\Omega, H)$ such that

$$L_\varepsilon(\omega)\xi_\varepsilon(\omega) + T(\omega)\xi_\varepsilon(\omega) = 0. \tag{4}$$

We have

$$\begin{aligned} 0 &= \big(L_\varepsilon(\omega)\xi_\varepsilon(\omega) + T(\omega)\xi_\varepsilon(\omega), \xi_\varepsilon(\omega)\big) \\ &= \big(L_\varepsilon(\omega)\xi_\varepsilon(\omega), \xi_\varepsilon(\omega)\big) + \big(T(\omega)\xi_\varepsilon(\omega), \xi_\varepsilon(\omega)\big), \end{aligned}$$

which implies that

$$\big(T(\omega)\xi_\varepsilon(\omega), \xi_\varepsilon(\omega)\big) \leqslant 0.$$

By the coercivity of T, there exists $M > 0$ such that $\|\xi_\varepsilon(\omega)\| \leqslant M$ for all $\varepsilon > 0$ and $\omega \in \Omega$. Setting $v_\varepsilon(\omega) = [I + \varepsilon L(\omega)]^{-1}\xi_\varepsilon(\omega)$, we have

$$\|v_\varepsilon(\omega)\| \leqslant \|\xi_\varepsilon(\omega)\| \leqslant M.$$

By (4), the boundedness of T, and the fact that $L_\varepsilon(\omega)\xi_\varepsilon(\omega) = L(\omega)v_\varepsilon(\omega)$, we conclude that there exists $M_1(\omega) > 0$ such that $\|L(\omega)v_\varepsilon(\omega)\| \leqslant M_1(\omega)$ for all $\varepsilon > 0$. Let $\{\varepsilon_n\}$ be a sequence of positive numbers such that $\varepsilon_n \to 0$. We set $\xi_n(\omega) = \xi_{\varepsilon_n}(\omega)$, $v_n(\omega) = v_{\varepsilon_n}(\omega)$. As in the proof of Theorem 1, there exists $\xi \in B(\Omega, H)$ such that for a fixed $\omega \in \Omega$, there is a subsequence of $\{\varepsilon_n\}$ (which we denote again by $\{\varepsilon_n\}$) such that

$$\xi_n(\omega) \rightharpoonup \xi(\omega), \qquad v_n(\omega) \rightharpoonup v(\omega), \quad \text{and} \quad L(\omega)v_n(\omega) \rightharpoonup u(\omega).$$

We have $\xi_\varepsilon(\omega) = v_\varepsilon(\omega) + \varepsilon L(\omega)v_\varepsilon(\omega)$. Letting $\varepsilon \to 0$, we get $\xi(\omega) = v(\omega)$. Since L is a weakly closed operator, $\xi(\omega) \in D$ and $L(\omega)\xi(\omega) = u(\omega)$.

By the monotonicity of L_n we have

$$(L_n(\omega)\xi_n(\omega) - L_n(\omega)\xi(\omega), \xi_n(\omega) - \xi(\omega)) \geqslant 0,$$

or

$$(-T(\omega)\xi_n(\omega) - L_n(\omega)\xi(\omega), \xi_n(\omega) - \xi(\omega)) \geqslant 0. \tag{5}$$

By [2, Proposition VII.2], we have $\|L_n(\omega)\xi(\omega)\| \leqslant \|L(\omega)\xi(\omega)\|$. Then from (5) we get $\limsup(T(\omega)\xi_n(\omega), \xi_n(\omega) - \xi(\omega)) \leqslant 0$. Since $T(\omega)\xi_n(\omega) \rightharpoonup -u(\omega)$ and T is of type (M), we conclude that $T(\omega)\xi(\omega) = -u(\omega)$. Thus, $0 = L(\omega)\xi(\omega) + T(\omega)\xi(\omega)$, and the proof is complete. ∎

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