ΑΠΑΝΤΗΣΕΙΣ ΠΡΟΒΛΗΜΑΤΩΝ

ΚΕΦΑΛΑΙΟ 9

9.1 Προβλήματα

1. $f(x) \sim \frac{\sinh 6}{12} + \sum_{n=1}^{\infty} \frac{(-1)^n \left(e^6 - e^{-6}\right)}{36 + n^2\pi^2}\left[3\cos\left(\frac{n\pi x}{3}\right) - \frac{n\pi}{6}\sin\left(\frac{n\pi x}{3}\right)\right]$

2. $f(x) \sim \frac{2\pi^2}{3} + \sum_{n=1}^{\infty}\left[\frac{4(-1)^n}{n^2}\cos(nx) + \frac{2(-1)^n}{n}\sin(nx)\right]$

3. $f(x) \sim -4$, $a_n = b_n = 0$, $n = 1, 2, \ldots$

4. $f(x) \sim \frac{1}{2} + \sum_{n=1}^{\infty}\left\{\frac{-2}{n\pi}\sin\left(\frac{n\pi}{2}\right)\cos\left(\frac{n\pi x}{2}\right) + \frac{4}{n\pi}\left[\cos\left(\frac{n\pi}{2}\right) - \cos(n\pi)\right]\sin\left(\frac{n\pi x}{2}\right)\right\}$

5. $f(x) \sim -1$, $a_n = b_n = 0$, $n = 1, 2, \ldots$ **6.** $f(x) \sim \frac{4\pi^2}{3} + \sum_{n=1}^{\infty}\left[\frac{4}{n}\cos(nx) - \frac{4\pi}{n}\sin(nx)\right]$

7. $f(x) \sim \frac{\pi^2}{6} + \sum_{n=1}^{\infty}\left\{\frac{2(-1)^n}{n^2}\cos(nx) + \left[(-1)^n - 1\right]\left(\frac{2}{\pi n^3} - \frac{(-1)^n \pi}{n}\right)\sin(nx)\right\}$

8. $f(x) \sim \frac{2}{\pi} - \sum_{n=1}^{\infty} \frac{4}{\pi\left(4n^2 - 1\right)}\cos(nx)$, $b_n = 0$, $n = 1, 2, \ldots$

9. $f(x) \sim \frac{1}{2} + \frac{1}{2}\cos(4x)$, $a_n = 0$, για $n \neq 0, 4$ και $b_n = 0$, $n = 1, 2, \ldots$.

10. $f(x) \sim 2\sin(3x)$, (δηλαδή η $f(x)$ είναι μια σειρά Fourier στο $[-\pi, \pi]$)

11. $f(x) \sim -\frac{1}{3}\sinh(3) + \sum_{n=1}^{\infty}\left\{\frac{6(-1)^{n+1}\sinh(3)}{9 + n^2\pi^2}\cos\left(\frac{n\pi x}{3}\right) + \left[\frac{-18}{n\pi} + \frac{2n\pi}{9 + n^2\pi^2}\sinh(3)\right](-1)^n \sin\left(\frac{n\pi x}{3}\right)\right\}$

12. $f(x) \sim -\frac{1}{2} + \sum_{n=1}^{\infty} \frac{6}{n^2\pi^2}\left[1 - \cos(n\pi)\right]\cos\left(\frac{n\pi x}{3}\right) \equiv$

$\equiv -\frac{1}{2} + \sum_{n=1}^{\infty} \frac{12}{(2n-1)^2\pi^2}\cos\left(\frac{(2n-1)\pi x}{3}\right)$

9.2 Προβλήματα

1. λεία

2. τίποτα από αυτά

3. λεία

4. τμηματικά λεία συνεχής

5. τμηματικά συνεχής

6. τμηματικά λεία συνεχής

7. συνεχής, όχι τμηματικά λεία

8. τίποτα από αυτά

9. ναι, $L = \frac{2\pi}{3}$

10. ναι, L = 1

11. ναι L = 6

12. ναι, L = π

13. ναι, L = 1/2

14. όχι

15. όχι

16. ναι, $L = \frac{\pi}{2}$

17. ναι, $L = \frac{\pi}{6}$

18. όχι

19. ναι $L = \frac{\pi}{2}$

20. ναι, L = 2π

21. άρτια

22. περιττή

23. άρτια

24. άρτια

25. περιττή

26. τίποτα

27. περιττή

28. άρτια

29. τίποτα

30. άρτια

31. περιττή

32. τίποτα

33. άρτια

34. άρτια

37. $g(x) = \frac{e^{x} + e^{-x}}{2} + \frac{e^{x} - e^{-x}}{2}$

38. $g(x) = x^{2}(1-x)^{-1} + x(1-x)^{2-1}$

39. $g(x) = 3 + (x^5 - 7x)$

40. $g(x) = \frac{2(1+x^2)}{1-x^2} + \frac{4x}{1-x^2}$

41. f(x) ≡ 0, στο IR.

45. (a) 1/2, (b) 5/2, (c) 2, (d) 1/2, (e) 1.

48. $\tilde{f}(x) = x^2, \quad 0 \le x < 3, \quad \tilde{f}(x) = (x-4)^2, \quad 3 < x \le 4, \quad \tilde{f}(3) = 5$

49. $\tilde{f}(x) = \sin x, \quad 0 < x < 2, \quad \tilde{f}(0) = \tilde{f}(2) = \frac{\sin 2}{2}$

50. $f(-2^{-}) = -4,\ f(-2^{+}) = 0,\ f(1^{-}) = 0,\ f(1^{+}) = 1,\ f(-3^{+}) = -6,\ f(3^{-}) = 9,$

$f'_{\alpha}(-2) = 0,\ f'_{\delta}(-2) = 0,\ f'_{\alpha}(1) = 0,\ f'_{\delta}(1) = 2,\ f'_{\delta}(-3) = 2,\ f'_{\alpha}(3) = 6$

$\tilde{f}(x) = x^2$, $1 < x < 3$, $\tilde{f}(x) = 0$, $-2 < x < 1$, $\tilde{f}(x) = 2x$, $-3 < x < -2$

$f(3) = f(-3) = \frac{3}{2}$, $f(1) = \frac{1}{2}$ και $f(-2) = -2$

51. $f(0^-) = 0$, $f(0^+) = 0$, $f'_\delta(0) = f'_\alpha(0) = 0$, $f\left(-\pi^+\right) = \pi^2$,

$f(\pi^-) = 2, f_\delta(-\pi) = -2\pi$, $f_\alpha(\pi) = 0$, $\tilde{f}(x) = x^2$, $-\pi < x < 0$, $\tilde{f}(x) = 2$, $0 < x < \pi$,

$f(0) = 1$, $f(-\pi) = f(\pi) = \frac{\pi^2 + 2}{2}$

52. $f(0^-) = 1$, $f(0^+) = 0$, $f'_\delta(0) = 1$, $f'_\alpha(0) = 0$,

$f\left(-\pi^+\right) = -1$, $f(\pi^-) = 0$, $f'_\delta(-\pi) = 0$, $f'_\alpha(\pi) = -1$,

$f(x) = \cos x$, $-\pi < x < 0$, $f(x) = \sin x$, $0 < x < \pi$, $f(0) = \frac{1}{2}$, $f(-\pi) = f(\pi) = -\frac{1}{2}$

53. $f(1^-) = 1$, $f(1^+) = e$, $f'_\delta(1) = e$, $f'_\alpha(1) = 1$,

$f(-2^+) = -2$, $f(2^-) = e^2$, $f'_\delta(-2) = 1$, $f'_\alpha(2) = e^2$

$\tilde{f}(x) = x$, για $-2 < x < 1$, $\tilde{f}(x) = e^x$, για $1 < x < 2$,

$\tilde{f}(1) = \frac{1+e}{2}$, $\tilde{f}(4) = \tilde{f}(-4) = \frac{e-2}{2}$

58. Οι συναρτήσεις: 55, 56, 57.

59. (i) $f(x) \sim \frac{\pi}{2} + \frac{4}{\pi}\sum_{n=1}^{\infty} \frac{\cos(2n-1)x}{(2n-1)^2}$, (ii) $f(x) \sim 2\sum_{n=1}^{\infty} \frac{\sin(nx)}{n}$,

(iii) $f(x) \sim \frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{\sin(2nx)}{n}$

60. (a) $f(x) \sim 1 + \frac{8}{\pi^2}\sum_{n=1}^{\infty} \frac{\cos\left(n-\frac{1}{2}\right)\pi x}{(2n-1)^2}$ (b) $f(x) \sim \frac{4}{\pi}\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} \sin\frac{n\pi x}{2}$

61. (a) $f(x) \sim \frac{2}{\pi} + \frac{4}{\pi}\sum_{n=1}^{\infty} \frac{\cos(2nx)}{1-4n^2}$, (b) $f(x) = \sin x$

62. (a) $f(x) \sim \frac{e^2-1}{2} + 4\sum_{n=1}^{\infty}\left(\frac{e^2(-1)^n - 1}{n^2\pi^2 + 4}\right)\cos(n\pi x)$

(b) $f(x) \sim 2\pi\sum_{n=1}^{\infty}\left(\frac{n\left[1 - e^2(-1)^n\right]}{n^2\pi^2 + 4}\right)\sin(n\pi x)$

63. (a) $f(x) \sim \frac{1}{\pi}\sinh\pi + \sum_{n=1}^{\infty}\frac{2}{\pi}\frac{1}{1+n^2}(-1)^n \sinh\pi\cos(nx)$

(b) $f(x) \sim \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{n}{1+n^2} \left[1 + (-1)^{n+1} \cosh\pi\right] \sin(nx)$

64. (a) $f(x) \sim 3$, (b) $f(x) \sim \sum_{n=1}^{\infty} \frac{12}{(2n-1)\pi} \sin[(2n-1)x]$

65. (a) $f(x) \sim \frac{\pi^2 - 2}{2\pi} + \sum_{n=1}^{\infty} \left\{ \frac{-4}{\pi n^2} \cos(n) - \frac{2}{\pi n} \sin(n) + \frac{2}{\pi n^2} \left[1 - (-1)^n\right] \right\} \cos(nx)$

(b) $f(x) \sim \frac{2}{\pi} \sum_{n=1}^{\infty} \left[-\frac{2}{n^2} \sin(n) + \frac{2}{n} \cos(n) - \frac{\pi}{n} (-1)^n \right] \sin(nx)$

66. (a) $f(x) \sim 3\sin(1) + 6\sin(1) \sum_{n=1}^{\infty} \frac{1}{1-n^2\pi^2} \cos(2nx)$

(b) $f(x) \sim \sum_{n=1}^{\infty} \frac{12(2n-1)\pi\cos(1)}{(2n-1)^2\pi^2 - 4} \sin[(2n-1)\pi x]$

67. (a) $f(x) \sim \frac{1}{2} + \frac{1}{2}\cos(2\pi x)$

(b) $f(x) \sim \frac{4}{3\pi}\sin(\pi x) + \sum_{n=3}^{\infty} \left\{ \frac{1-(-1)^n}{n\pi} + \frac{1-(-1)^n}{(n^2-4)\pi} \right\} \sin(n\pi x)$

68. (a) $f(x) \sim 1 - \sum_{\substack{k=1 \\ k \neq 2m}}^{\infty} \frac{4}{k^2\pi^2} \cos(k\pi x)$

(b) $f(x) \sim 8 \sum_{k=1}^{\infty} \left\{ \frac{\sin\left[(4k+1)\frac{\pi x}{2}\right]}{(4k+1)^2\pi^2} + \frac{\sin\left[(4k+3)\frac{\pi x}{2}\right]}{(4k+3)^2\pi^2} \right\}$

69. (a) $f(x) \sim \frac{2}{3} + \sum_{k=1}^{\infty} \left\{ \frac{\sqrt{3}}{(3k+2)\pi} \cos\frac{(6k+4)\pi x}{3} - \frac{\sqrt{3}}{(3k+1)\pi} \cos\frac{(6k+2)\pi x}{3} \right\}$

(b) $f(x) \sim \sum_{k=1}^{\infty} \left\{ \frac{2}{(6k+1)\pi} \sin\frac{(6k+1)\pi x}{3} - \frac{4}{(6k+3)\pi} \sin[(2k+1)\pi x] \right\}$

9.3 Προβλήματα

1. $\frac{\pi^2}{6} = 1,6449341\ldots$. **2.** $\frac{\pi^4}{90} = 1,0823\ldots$

3. (β) $\left[\frac{\pi\cos(a\pi)}{a\sin(a\pi)} - \frac{1}{a^2}\right] / 2$ (γ) $\left[\frac{\pi^2}{a^2\sin^2(a\pi)}\left[1 + \frac{\sin(2a\pi)}{2a\pi}\right] - 2a^{-4}\right] / 4$

9.4 Προβλήματα

1. (a) $x^2 \sim \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}\left[\frac{(-1)^n}{n^2}\right]\cos(nx)$ (b) $x^4 \sim \frac{\pi^4}{5} + \sum_{n=1}^{\infty}\left[48\frac{(-1)^{n+1}}{n^4} + 8\pi^2\frac{(-1)^n}{n^2}\right]\cos(nx)$

2. $f'(x) \sim \sum_{n=1}^{\infty}\left\{\left(\frac{2[(-1)^n - 1]}{n^2\pi^2}\right)\cos\left(\frac{n\pi x}{2}\right) + \left[\frac{2(-1)^{n+1}}{n\pi}\right]\sin\left(\frac{n\pi x}{2}\right)\right\}$

3. $y(x) = 1 + \sum_{n=1}^{\infty}\left\{\frac{La_n}{n\pi}\sin\left(\frac{n\pi x}{L}\right) - \frac{Lb_n}{n\pi}\left(\cos\left(\frac{n\pi x}{L}\right) - 1\right)\right\}$

9.5 Προβλήματα

1. $f(x,y) \sim \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} 4\left[\frac{2 - n^2\pi^2}{n^3}(-1)^n - \frac{2}{n^3}\right]\frac{(-1)^{m+1}}{m}\sin(nx)\sin(mx)$

2. $f(x,y) \sim \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} 4mn\pi^2\,\frac{e^2(-1)^{m+1}+1}{4+m^2\pi^2}\,\frac{e^4(-1)^{n+1}+1}{16+n^2\pi^2}\sin\frac{n\pi x}{4}\sin\frac{m\pi y}{2}$

3. $f(x,y) \sim \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{16(-1)^n(-1)^m}{n\left(4+m^2\pi^2\right)}\,m\sinh 2\sin\frac{n\pi x}{4}\sin\frac{m\pi y}{2}$

4. $f(x,y) \sim \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{-8}{m}\left[\frac{\sin(8-n\pi)}{8-n\pi} - \frac{\sin(8+n\pi)}{8+n\pi}\right](-1)^m\sin\frac{n\pi x}{4}\sin my$

5. $f(x,y) \sim 2\pi^2 + \sum_{m=1}^{\infty}\frac{8}{m^2}\left[(-1)^m - 1\right]\cos\frac{my}{2} + \sum_{n=1}^{\infty}\frac{8}{n^2}\left[(-1)^n - 1\right]\cos\frac{nx}{2}$
$+ \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{16}{n^2m^2\pi^2}\left[(-1)^n - 1\right]\left[(-1)^m - 1\right]\cos\frac{nx}{2}\cos\frac{my}{2}$

6. $f(x,y) \sim \frac{\pi}{9} + \sum_{m=1}^{\infty}\frac{4}{3m^2}(-1)^m\cos m\pi y + \sum_{n=1}^{\infty}\frac{4}{3n^2}(-1)^n\sin nx$
$+ \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{16}{n^2m^2\pi^2}(-1)^{n+m}\cos nx\cos m\pi y$

7. $f(x,y) \sim 2 + \sum_{n=1}^{\infty}\frac{8}{n^2\pi^2}\left[(-1)^n - 1\right]\cos\frac{n\pi x}{4}$

8. $f(x,y) \sim \frac{100}{9} + \sum_{n=1}^{\infty}\frac{400(-1)^n}{3m^2\pi^2}\cos\frac{m\pi y}{5}$
$+ \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{1600}{n^2m^2\pi^2}(-1)^n(-1)^m\cos\frac{n\pi x}{2}\cos\frac{m\pi y}{5}$

9. $f(x,y) \sim \sum_{n=1}^{\infty} \frac{4\sinh(1)}{n\pi}(-1)^{n+1}\sin\frac{n\pi x}{2}$

$$+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{8(-1)^n(-1)^m}{n\pi(1+m^2\pi^2)}\sinh(1)\left[-\sin\frac{n\pi x}{2}\cos(m\pi y)+m\pi\sin\frac{n\pi x}{2}\sin(m\pi y)\right]$$

10. $f(x,y) \sim \frac{\pi}{9} + \frac{1}{2}\sum_{m=1}^{\infty}\frac{8}{3}\frac{(-1)^m}{\pi^2 m^2}\cos mx + \frac{1}{2}\sum_{n=1}^{\infty}\frac{8}{3}\frac{(-1)^n}{\pi^2 n^2}\cos ny$

$$+\frac{1}{2}\sum_{n=1}^{\infty}\frac{8}{3}\pi^2\frac{(-1)^n}{n^2}\cos(ny)$$

11. $f(x,y) \sim \sum_{m=1}^{\infty}\frac{2(-1)^{m+1}}{m}\sin(mx)\sin(y)$

9.6 Προβλήματα

1. $f(x) \sim \frac{100}{\pi}\int_0^{\infty}(\sin(2-x)\alpha + \sin\alpha x)\frac{d\alpha}{\alpha}$

2. $f(x) \sim \frac{1}{\pi}\int_0^{\infty}\frac{\cos\alpha x + \alpha\sin\alpha x}{1+\alpha^2}d\alpha$ **3.** $f(x) \sim \frac{2}{\pi}\int_0^{\infty}\left(\frac{1-\cos\alpha}{\alpha}\right)\sin\alpha x\, d\alpha$

4. $f(x) \sim \frac{4b}{\pi c^2}\int_0^{\infty}\frac{1}{\alpha^3}[\sin(\alpha c) - \alpha c\cos(\alpha c)]\cos(\alpha x)d\alpha$

5. $f(x) \sim \frac{4}{\pi}\int_0^{\infty}\left(\frac{\sin\alpha - \alpha\cos\alpha}{\alpha^3}\right)\cos\alpha x\, d\alpha$

6. $f(x) \sim \int_0^{\infty}\frac{\cos x\alpha + \alpha\sin x\alpha}{1+\alpha^2}d\alpha$ **7.** $f(x) \sim \int_0^{\infty}\frac{\cos\left(\frac{\pi\alpha}{2}\right)\cos(\alpha x)}{1-\alpha^2}d\alpha$

8. $f(x) \sim \frac{2}{\pi}\int_0^{\infty}\frac{\alpha^3\sin\alpha x}{\alpha^4+4}d\alpha$ **9.** $f(x) \sim \int_0^{\infty}\frac{\sin(\pi\alpha)\sin(x\alpha)}{1-\alpha^2}d\alpha$

10. $f(x) \sim \frac{2}{\pi}\int_0^{\infty}\left[\left(b^2 - \frac{2}{b^2}\right)\sin\alpha b + \frac{2b}{\alpha}\cos\alpha b\right]\frac{\cos\alpha x}{\alpha}d\alpha$

11. $f(x) \sim \frac{2}{\pi}\int_0^{\infty}\frac{1}{\alpha}\cos\frac{\alpha(b+c-2x)}{2}\sin\frac{\alpha(c-b)}{2}d\alpha$

12. $f(x) \sim \frac{2}{\pi}\int_0^{\infty}\frac{1}{\alpha}[\cos(\alpha b) - \cos(\alpha c)]\sin(\alpha x)d\alpha$

$$f(x) \sim -\frac{2}{\pi}\int_0^{\infty}\frac{1}{\alpha}[\sin(\alpha b) - \sin(\alpha c)]\cos(\alpha x)d\alpha$$

13. $f(x) \sim \frac{2b}{\pi c}\int_0^{\infty} \frac{1-\cos \alpha c}{\alpha^2}\cos(\alpha x)\,d\alpha$ **14.** $f(x) \sim \int_0^{\infty} \frac{2\alpha}{\pi(1+\alpha^2)}\sin(\alpha x)\,d\alpha$

15. $I = \frac{3\pi}{16}$

9.7 Προβλήματα

1. $f_S(n) = \pi(-1)^n\left(\frac{6}{\pi^3} - \frac{\pi^2}{n}\right)$ **2.** $f_S(n) = \frac{n}{n^2+1}\left[1-(-1)^n e^{\pi}\right]$

3. $f_S(n) = \begin{cases} \frac{n}{n^2-a^2}\left[1-(-1)^n\cos(a\pi)\right], & \gamma\iota\alpha \quad a \notin N \\ 0, & \gamma\iota\alpha \quad a \in \mathbb{N} \end{cases}$

4. $f_S(n) = \frac{n}{n^2+1}\left[1-(-1)^n e^{-\pi}\right]$ **5.** $f(n) = \begin{cases} \frac{2\pi(-1)^n}{n^2}, & \gamma\iota\alpha \quad n = 1, 2, 3, \ldots \\ \frac{\pi}{3}, & \gamma\iota\alpha \quad n = 0 \end{cases}$

6. $f_C(n) = \frac{(-1)^n e - 1}{1+n^2}$ **7.** Αν $a = k \in \mathbf{N}$, $f_C(n) = \begin{cases} \frac{\pi}{2}, & \gamma\iota\alpha \quad n = k, \\ 0, & \gamma\iota\alpha \quad n = 0. \end{cases}$

9.8 Προβλήματα

1. $\mathcal{F}[f(x)] = F(\alpha) = \sqrt{\frac{2}{\pi}}\frac{b}{\alpha^2+b^2}$ **2.** $\mathcal{F}[f(x)] = F(\alpha) = \frac{n!}{\sqrt{2}(b-i\alpha)^{n+1}}$

3. $\mathcal{F}[f(x)] = F(\alpha) = \sqrt{\frac{2}{\pi}}\,\frac{2}{\alpha}e^{i\alpha(b+c)/2}\sin\frac{\alpha(c-b)}{2}$

4. $\mathcal{F}[f(x)] = F(\alpha) = \frac{4}{\sqrt{2\pi}}\,\frac{c}{\alpha^2 b}\sin^2\left(\frac{b\alpha}{2}\right)$

5. $\mathcal{F}[f(x)] = F(\alpha) = \frac{1}{\sqrt{2\pi}}\left[\frac{-4c}{\alpha^2 b}\cos(\alpha b) + \frac{4c}{\alpha^3 b^2}\sin(\alpha b)\right]$

6. $\mathcal{F}[f(x)] = F(\alpha) = \sqrt{\frac{2}{\pi}}\,\frac{\cos\left(\frac{\pi\alpha}{2}\right)}{1-\alpha^2}e^{i\pi\alpha/2}$ **7.** $\mathcal{F}[f(x)] = F(\alpha) = i\sqrt{\frac{2}{\pi}} - \frac{1-\cos a}{\alpha}$

8. $\mathcal{F}[f(x)] = \frac{1}{\sqrt{2\pi}}\left[\frac{\alpha}{1+\alpha^2}\sinh L\cos L(a) + \frac{2\alpha}{1+\alpha^2}\cosh L\sin L(a)\right]$

9. $\mathcal{F}[f(x)] = F(\alpha) = \frac{1}{\sqrt{2\pi}}\,\pi e^{-|\alpha|}$ **10.** $F(\alpha) = \sqrt{\frac{1}{2}}\cos\left(\frac{\alpha^2}{4} + \frac{\pi}{4}\right)$

11. $F(\alpha)=\sqrt{\frac{1}{2}}\cos\left(\frac{\alpha^2}{4}-\frac{\pi}{4}\right)$ **12.** $F(\alpha)=\frac{1}{\sqrt{2\pi}}\left[\frac{2}{1+\alpha^2}-\frac{1}{1+(\alpha-2)^2}-\frac{1}{1+(\alpha+2)^2}\right]$

13. $F(\alpha)=\exp\left[-\frac{1}{2}\left(\alpha+\frac{3i}{2}\right)^2\right]$ **14.** $F(\alpha)=\sqrt{\frac{1}{2}}\left\{\exp\left[-\frac{1}{2}(1+\alpha)^2\right]+\exp\left[-\frac{1}{2}(1-\alpha)\right]^2\right\}$

15. $F_C(\alpha)=\frac{1}{\alpha}\sqrt{\frac{2}{\pi}}\left[\sin(\alpha c)-\sin(\alpha b)\right]$, $F_S(\alpha)=\frac{1}{\alpha}\sqrt{\frac{2}{\pi}}\left[\cos(\alpha b)-\cos(\alpha c)\right]$

16. $F_C(\alpha)=\frac{4c}{\alpha^2 b}\sqrt{\frac{2}{\pi}}\cos(\alpha d)\sin^2\left(\frac{\alpha b}{2}\right)$, $F_S(\alpha)=\frac{4c}{\alpha^2 b}\sin(\alpha d)\sin^2\left(\frac{\alpha b}{2}\right)$

17. $F_C(\alpha)=\sqrt{\frac{2}{\pi}}\frac{1-\alpha^2}{(1+\alpha^2)^2}$, $F_S(\alpha)=\sqrt{\frac{2}{\pi}}\frac{\alpha}{(1+\alpha^2)^2}$

18. $F_C(\alpha)=\frac{2L\cos(\alpha L)}{\alpha^2}+\frac{L^2\alpha^2-2}{\alpha^3}\sin(\alpha L)$ **19.** $F_C(\alpha)=\frac{b^2-\alpha^2}{(b^2+\alpha^2)^2}$

20. $F_C(\alpha)=\frac{\sqrt{\pi}}{2\sqrt{b}}e^{-\alpha^2/4b}$ **21.** $F_C(\alpha)=\begin{cases}\frac{\pi}{2}e^{-\alpha}, & \alpha>0\\ \frac{\pi}{2}e^{\alpha}, & \alpha<0\end{cases}$

22. $F_S(\alpha)=\frac{2\alpha}{(1+\alpha^2)^2}$ **23.** $F_S(\alpha)=\frac{1}{2}\left[\frac{1}{1+(1-\alpha)^2}-\frac{1}{1+(1+\alpha)^2}\right]$

24. $F_S(\alpha)=\begin{cases}0, & \text{για } 0<\alpha<1,\\ \frac{\pi}{4}, & \text{για } \alpha=1,\\ \frac{\pi}{2}, & \text{για } \alpha>1.\end{cases}$