ΑΠΑΝΤΗΣΕΙΣ ΠΡΟΒΛΗΜΑΤΩΝ

ΚΕΦΑΛΑΙΟ 7

7.2 Προβλήματα

1. ΤΣ	**2.** Σ	**3.** Α
4. ΤΣ	**5.** Σ	**6.** Α
7. c=2	**8.** όχι	**9.** c>1
10. c=1	**11.** c=1	**12.** c=1
13. c=1	**14.** c=0	**15.** c=1
16. c=1	**17.** c=0	**18.** c=a
19. όχι	**20.** c=2	**21.** c=1

7.3 Προβλήματα

1. $F(s) = \dfrac{3s^2 + 2s + 3}{(s^2+1)^2}$

2. $F(s) = \sqrt{\pi}\left(2s^{3/2}\right)^{-1} + 3\sqrt{\pi} / \left(2s^{5/2}\right)$

3. $F(s) = \dfrac{a}{s^2 - a^2}$

4. $F(s) = \ln\left(\dfrac{s}{\sqrt{s^2 + a^2}}\right)$

5. $F(s) = \ln\left(\dfrac{s-b}{s-a}\right)$

6. $F(s) = (s^2 + 2a^2)\left[s\left(s^2 + 4a^2\right)\right]^{-1}$

7. $F(s) = 6a^3\left[(s^2 + a^2)(s^2 + 9a^2)\right]^{-1}$

8. $F(s) = 2a^2 s\left[(s^2 + a^2)(s^2 + 9a^2)\right]^{-1}$

9. $F(s) = 2(s-a)^{-3}$

10. $F(s) = 2s\left(s^2 - 3b^2\right)\left(s^2 + b^2\right)^{-3}$

11. $F(s) = 6(s-a)^{-4}$

12. $F(s) = (s^2 + a^2)(s^2 - a^2)^{-2}$

13. $F(s) = 2\,sa\,(s^2 - a^2)^{-2}$

14. $F(s) = s^3(s^4 + 4a^4)^{-1}$

15. $F(s) = a\left(s^2 + 2a^2\right)\left(s^4 + 4a^4\right)^{-1}$

16. $F(s) = a\left(s^2 - 2a^2\right)\left(s^4 + 4a^4\right)^{-1}$

17. $F(s) = 2a^2 s\left(s^4 + 4a^4\right)^{-1}$

18. $F(s) = n\pi\left[(s+2)^2 + n^2\pi^2\right]^{-1}$

19. $F(s) = \left[A(s+a) + B\beta\right]\left[(s+a)^2 + \beta^2\right]^{-1}$

20. $F(s) = \dfrac{1}{s} - \dfrac{4}{s^2} + \dfrac{4}{(s+3)^3}$

21. $F(s) = \dfrac{6s}{(s^2+9)^2} - \dfrac{1}{s} + \dfrac{4}{s^3}$

22. $F(s) = 2^{-1}\left[(s-6)^{-1} + (s+6)^{-1} + 2s^{-1}\right]$

23. $F(s) = 3(s-3)^{-2} - 3(s+3)^{-2} - 54\, s^{-4}$

24. $F(s) = 4(s+1)^{-3} - (2s)^{-1} + s\left[2(s^2+4)\right]^{-1}$

25. $F(s) = s\left[2(s^2-16)\right]^{-1} - s\left[2(s^2+16)\right]^{-1}$

26. $F(s) = (s^2-9)(s^2+9)^{-2} - s(s^2+9)^{-1} + 5(s+1)^{-1}$

27. $F(s) = (s-3)^{-1} - 12\, s^{-4}$

28. $F(s) = s^{-1} - 6(s^2-36)^{-1} + (s+1)\left[(s+1)^2 - 1\right]^{-1}$

29. $F(s) = 6\left[(s+4)^2+4\right]^{-1} - s(s^2+36)^{-1} + 6s^{-1}$

30. $F(s) = 2(s-3)^{-1}\left(\dfrac{1-e^{-(s-3)}}{1+e^{-(s-3)}}\right)$

31. $F(s) = \dfrac{h}{s}\left(\dfrac{1-e^{-4s}}{1+e^{-4s}}\right)$

32. $F(s) = (1+e^{-\pi s})\left[(1-e^{-2\pi s})(s^2+1)\right]^{-1}$

33. $F(s) = (1-e^{-s})^2\left[s(1-e^{-2s})\right]^{-1}$

34. $F(s) = (1+e^{-\pi s})\left[(1-e^{-\pi s})(s^2+1)\right]^{-1}$

35. $F(s) = \dfrac{e^{1-s}-1}{(1-s)(1-e^{-s})}$

36. $F(s) = \dfrac{1}{1-e^{-\pi s}}\left[\dfrac{2-e^{-\pi s/2}(\pi s+2)}{\pi s^2} + \dfrac{se^{-\pi s/2}+e^{-\pi s}}{s^2+1}\right]$

37. $F(s) = \dfrac{1}{as^2}\tanh\left(\dfrac{as}{2}\right)$

38. $F(s) = \dfrac{a}{s}\left(\dfrac{1}{bs} - \dfrac{1}{e^{bs}-1}\right)$

39. $G(s) = a\left[s(e^{bt}-1)\right]^{-1}$

40. (a) $\left(1-(2s^2)^{-1} + 1\cdot 3\left[1\cdot 2\, s^4\right]^{-1} - \ldots\right) = \left(1+s^2\right)^{-1/2}$

(c) $\mathcal{L}\{\mathrm{erf}(t)\} = \dfrac{2}{\sqrt{\pi}}\sum_{k=0}^{\infty}\dfrac{(-1)^k(2k)!}{k!\, s^{2k+2}}$ (d) $\left[s(s+1)^{1/2}\right]^{-1}$

42. (i) $F(3i) = -\dfrac{2}{5} = \overline{F(-3i)}$ (ii) $F(a+ib) = \dfrac{2}{(a+ib)^2+4} = \overline{F(a-ib)}$

7.4 Προβλήματα

1. $\dfrac{1}{3^{3/2}}\left[\sin(\sqrt{3}\,t) - \sqrt{3}\,t\cos(\sqrt{3}\,t)\right] - \dfrac{1}{180}t^6$

2. $\dfrac{1}{6}e^{3t} - \dfrac{1}{6}e^{-3t} - 7\cos(\sqrt{15}\,t)$

3. $\dfrac{t^3}{6} - \dfrac{t^4}{12} + \dfrac{t^5}{30}$

4. $\dfrac{2}{3}e^{3t} + \dfrac{1}{3}e^{-3t} - 1$

5. $\cos(\sqrt{6}\,t) - \dfrac{5}{\sqrt{6}}\sin(\sqrt{6}\,t)$

6. $-\dfrac{3}{2}t^2e^{-2t} + 4\cos(\sqrt{6}\,t)$

7. $\frac{4}{3}\sin(3t) - t\,e^{3t}$

8. $\frac{4}{3}\left(e^{2t} - e^{-t}\right)$

9. $e^t - 2t\,e^t$

10. $\frac{7}{3}e^{-2t} - \frac{1}{3}e^t$

11. $t^2 e^t \left(1 - \frac{1}{3}t\right)$

12. $e^{2t}\cos\left(\sqrt{15}\,t\right)$

13. $\frac{2}{3}e^t + \frac{1}{3}\cos\left(\sqrt{2}\,t\right) - \frac{1}{3\sqrt{2}}\sin\left(\sqrt{2}t\right)$

14. $-\frac{7209}{882}e^{2t} - \frac{1890}{441}t\,e^{2t} + \frac{12299}{882}e^{4t} + \frac{983}{441}e^{-5t}$

15. $\sin\left(\frac{2\,n\pi\,t}{T}\right)$

16. $\frac{t^{3/2}}{\Gamma(1/2)}$

17. $\frac{1}{2\,\omega^3}\{\sin\omega t - \omega t\cos\omega t\}$

18. $\frac{1}{2\omega}\{\sin\omega t + \omega t\cos\omega t\}$

19. $e^{-t}\frac{\sin t}{t}$

20. $\frac{e^t - 1}{t}$

21. $\frac{e^{-bt} - e^{-at}}{t}$

22. $\frac{4t\sin 2t + 3\sin 2t - 6t\cos 2t}{16}$

23. $t^2 e^{-3t}\left(1 + \frac{t}{6}\right)$

24. $7\cos 3t + 4\sin 3t$

27. $\mathcal{L}\{f_1(t)\} = \mathcal{L}\{f_2(t)\} = \mathcal{L}\{f_3(t)\} = s^{-2}, \qquad \mathcal{L}^{-1}\{s^{-2}\} = f_3(t).$

28. $\mathcal{L}\{f_1(t)\} = \mathcal{L}\{f_2(t)\} = \mathcal{L}\{f_3(t)\} = (s-1)^{-1}, \quad \mathcal{L}^{-1}\{(s-1)^{-1}\} = f_3(t).$

30. $\mathcal{L}^{-1}\{F(s)\} = \frac{4}{3} - \frac{e^{-t}}{8} - \frac{7}{4}e^t + \frac{13}{14}e^{3t}$

31. $\mathcal{L}^{-1}\{F(s)\} = \frac{e^{-t}}{4} - \frac{5}{4}e^{3t}$

32. $\mathcal{L}^{-1}\{F(s)\} = -\frac{2}{3} + \frac{3\,e^t}{4} - \frac{e^{-3t}}{12}$

33. $\mathcal{L}^{-1}\{F(s)\} = \frac{5\,e^{-t}}{2} - 9\,e^{-2t} + \frac{15\,e^{-3t}}{2}$

34. $\frac{3}{20}e^t - \frac{1}{4}e^{-t} + \frac{\cos t\,e^{-2t}}{10} - \frac{\sin t\,e^{-2t}}{5}$

36. $f(t) = 2\,e^{-t} - 4\,e^{3t} + 5\,e^{2t}$

37. $f(t) = 1 - 3\,e^{-t} + 3\,e^{-2t}$

38. $f(t) = \frac{(3 - 10t)\,e^{t}}{50} + \frac{e^{-t}\left(-9\cos 3t + 13\sin 3t\right)}{150}$

39. $f(t) = \frac{1}{25}\left(3\cos t + 4\sin t - 3\,e^{-2t} - 10\,t\,e^{-2t}\right)$

40. $f(t) = \left(\frac{4}{25} + \frac{2t}{5} + \frac{2\,t^2}{5} + \frac{t^3}{6}\right)\frac{e^{-2t}}{5^4} + \left(-\frac{4}{25} + \frac{2t}{5} - \frac{2\,t^2}{5} + \frac{t^3}{6}\right)\frac{e^{3t}}{5^4}$

7.5 Προβλήματα

1. $N = -\frac{1}{144} + \frac{t}{12} + e^{-t/2}\left(\cos\frac{\sqrt{47}\,t}{2} - \left(\frac{23}{\sqrt{47}}\right)\sin\left(\frac{\sqrt{47}\,t}{2}\right)\right) / 144$

2. $y = -t/7 + \frac{4}{147} + 3\,e^{7t}/490 - e^{-3t}/30$

3. $y = -e^{-2t}/9 + 4\,e^{t}/9 + (2/3)\,e^{-t/2}\left(\cos\sqrt{3}t/2\right) - \left(1/\sqrt{3}\right)\sin\left(\sqrt{3}t/2\right)\right)$

4. $x = e^{-t/2}\left(\cos\left(\sqrt{15}t/2\right) - \left(1/\sqrt{15}\right)\sin\left(\sqrt{15}t/2\right)\right)$

5. $y = -e^{t}\left(\cos h\left(\sqrt{2}t\right) + e^{t}\sin h\left(\sqrt{2}t\right)\right)/\sqrt{2}$

6. $y = \frac{19}{51}e^{4t} + \frac{11}{15}e^{-2t} - \frac{9}{85}\cos t - \frac{2}{85}\sin t$

7. $y = -\frac{t}{2}\cos t + \frac{1}{2}\sin t$

8. $R = \frac{1}{5}\left[e^{-t}\left(\cos h\,2t + \frac{3}{2}\sin h\,2t\right) + 3\sin t - \cos t\right]$

9. $y = \frac{43}{10}e^{-t} - \frac{29}{13}e^{-2t} - \frac{9}{130}\cos 3t - \frac{7}{130}\sin 3t$

10. $y = -\frac{e^{-t}}{10}\left[2\cos t + \sin t\right] + \frac{6}{5}\cos h\,2t - \frac{1}{20}\sin h\,2t$

11. $y = 2\,e^{2t} - 3\,e^{-t} - e^{-t}\sin 3t + e^{-t}\cos 3t$

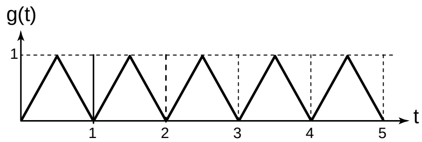
12. $y = e^{-2t} - e^{-3t} - 2te^{-3t}$

13. $y = -4te^{t} + 3e^{t} - 3\cos t + \sin t$

14. $y = -\frac{2e^{t}}{5}\cos t + \frac{e^{t}}{5}\sin t + \frac{\sin t}{5} + \frac{2\cos t}{5}$

15. $y = \int_0^t (t-\tau)\,e^{-2(t-\tau)}\,g(\tau)\,d\tau,$

όπου g η ***τριγωνική κυματική συνάρτηση*** του Σχήματος 1.)



Σχήμα 4 *Τριγωνική κυματική συνάρτηση*

16.
$y_1 = e^{t} + e^{2t}$
$y_2 = e^{2t}$

17.
$y_1 = e^{t}$
$y_2 = e^{-t}$
$y_3 = e^{t} - e^{-t}$

18.
$x = 8\sin t + 2\cos t$
$y = -13\sin t + \cos t + (e^{t} - e^{-t})/2$

19.
$x = e^{-2t} - t\,e^{t}$
$y = \left(e^{t} - e^{-2t} + 3t\,e^{t}\right)/3$

20.
$x = \left(-e^{t} + e^{-t} + 4\,e^{2t}\right)/2$
$y = \left(e^{t} + e^{-t} - 2\,e^{2t}\right)/2$

21.
$x = \left(3 - 3\,e^{2t/3} + 2t^{2} + 2t\right)/4$
$y = \left(-3\,e^{2t/3} + 2t + 3\right)/2$

22.
$x = \left(-1 + 1\,e^{-2t}\right)/2$
$y = \left(1 + 2t - 1e^{-2t}\right)/4$

23.
$x = -e^{-t}\cos(2t) - 32\,e^{-t}\sin(2t)$
$y = 7e^{-t}\cos(2t) + 19\,e^{-t}\sin(2t) - 6$

24. $x = \left(-21 - 18t + e^{-2t} + 20\, e^{t}\right)/6$
$y = \left(-3 + e^{-2t} + 2e^{t}\right)/3$

25. $x(t) = -\frac{7}{9} - \frac{1}{3}t - \frac{31}{99}e^{-3t/5} + \frac{108}{99}e^{t/2}$
$y(t) = -\frac{13}{9} - \frac{1}{3}t + \frac{62}{99}e^{-3t/5} + \frac{81}{99}e^{t/2}$

26. $x(t) = \begin{pmatrix} 1-t \\ t \\ t \end{pmatrix} e^{2t}$

27. $x(t) = \begin{pmatrix} t - t^2 - \frac{t^3}{6} - \frac{t^4}{2} + \frac{t^5}{2} \\ -t^2/2 \\ t^2 + t^3/6 \end{pmatrix}$

28. $x(t) = \begin{pmatrix} 1 \\ 1+t \\ 1 \\ 1+2t \end{pmatrix} e^{3t}$

29. $x(t) = 2\, e^{t} \begin{pmatrix} t\cos t + 3t\sin t + \sin t \\ -t\sin t \end{pmatrix}$

33. $y(x) = \frac{w_0}{24\, E\, I}\, x^2\, (x - L)^2$

34. $y(x) = \frac{w_0}{E\, I}\left(\frac{L^2}{4}x^2 - \frac{L}{6}x^3 + \frac{1}{24}x^4\right)$

35. (i) $x(t) = a_1\, e^{-Bt/2M} \cos\left(\sqrt{(\kappa/M) - B^2/(4M^2)}\; t\right)$

$$+ \frac{2\, a_2\, M + B\, a_1}{\sqrt{4M\kappa - B^2}}\, e^{-Bt/2M} \sin\left(\sqrt{(\kappa/M) - B^2/(4M^2)}\; t\right)$$

36. $I_1 = -e^{-20t} - 2\, e^{-5t} + 3$, $\quad I_2 = -2\, e^{-20t} + 2\, e^{-5t}$, $\quad I_3 = e^{-20t} - 4\, e^{-5t} + 3$

7.6 Προβλήματα

1. $f(t) = H_1(t) + H_3(t)$

2. $f(t) = 5 + 2(t-3)\, H_3(t)$

3. $f(t) = e^{-t} - e^{-t}\, H_2(t)$

4. $f(t) = 4\left[H_2(t) - H_5(t)\right]$

5. $f(t) = \cos t - \cos t\, H_{\pi/2}(t)$

6. $f(t) = t\left[H_1(t) - H_2(t)\right]$

7. $f(t) = 4 - 2\, H_1(t) - 2\, H_2(t)$

8. $f(t) = (t-1)\, H_1(t) + H_1(t) - t$

9. $f(t) = e^{6}\, H_2(t)\, e^{3(t-2)}$

11. $F(s) = 48\, \frac{e^{-2s}}{s}$

12. $F(s) = e^{-4s}\left\{\frac{128}{s} + \frac{96}{s^2} + \frac{48}{s^3} + \frac{12}{s^4}\right\}$

13. $F(s) = -\frac{e^{-5s}}{s^2} - \frac{3e^{-5s}}{s} + \frac{2}{s^2}$

14. $F(s) = \frac{h}{s}\left(\frac{1 - e^{-4s}}{1 + e^{-4s}}\right)$

15. $F(s) = \frac{1000\, e^{-5s}}{s} + \frac{600\, e^{-5s}}{s^2} + \frac{240\, e^{-5s}}{s^3} + \frac{48\, e^{-5s}}{s^4} - e^{-3(s+4)}\frac{3s + 13}{(s+4)^2}$

16. $F(s) = \frac{1}{s^2} - \frac{37\,e^{-s}}{s} - \frac{14\,e^{-s}}{s^2} - \frac{14\,e^{-s}}{s^3}$

17. $F(s) = \frac{6320\,e^{-9s}}{s} + \frac{2862\,e^{-9s}}{s^2} + \frac{966\,e^{-9s}}{s^3} + \frac{216\,e^{-9s}}{s^4} + \frac{24\,e^{-9s}}{s^5}$

18. $F(t) = \frac{3}{(s+2)^2 + 9}\,e^{-s}$

19. $F(s) = \left(\frac{s^2 + s + 2}{s^3}\right) e^{-2s}$

20. $F(s) = 2\,e^{-s} / s^3$

21. $F(s) = \frac{e^{3-3s}}{s-1}$

22. $F(s) = \frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-3s}}{s^2} + \frac{e^{-4s}}{s^2}$

23. $F(s) = \frac{1}{s^2} - \frac{2\,e^{-2s}}{s^2} + \frac{e^{-4s}}{s^2}$

24. $F(s) = \frac{1}{s^2} - \frac{2\,e^{-s}}{s^2\left(1 + e^{-2s}\right)}$

25. $F(s) = \frac{1}{s^2} - \frac{2\,e^{-s}}{s^2} + \frac{e^{-3s}}{s^2} + \frac{e^{-4s}}{s}$

26. $F(s) = \frac{1}{s} + \sum_{n=0}^{\infty} (-1)^n\,e^{-ns} = \frac{1/s}{1 + e^{-s}}, \quad s > 0$

27. $F(s) = \frac{1 - (1+s)\,e^{-s}}{s^2\,(1 - e^{-s})}$

28. $F(s) = \frac{1 + e^{-s\pi}}{\left(1 + s^2\right)(1 - e^{-s\pi})}, \quad s > 0$

29. $f(t) = \frac{1}{2}\,H_1(t)\sin(2t - 1)$

30. $f(t) = \left(\frac{e^{-t}}{\sqrt{5}}\right) \sin\sqrt{5}\,t - H_\pi(t)\left(\frac{e^{-(t-\pi)}}{\sqrt{5}}\right) \sin\sqrt{5}\,(t - \pi)$

31. $f(t) = H_{0.5}(t)\left(-e^{-5\,(t - 1/2)} + e^{(t - 1/2)}\right)/6$

32. $f(t) = \frac{H_2(t)}{7} + \frac{3}{7} - \frac{1}{7} H_2(t)\,e^{-2(t-2)}\left[\cos\sqrt{3}\,(t-2) + \frac{2}{\sqrt{3}}\sin\sqrt{3}\,(t-2)\right] - \frac{3}{7}\,e^{-2t}\left[\cos\sqrt{3}\,t + \frac{2}{\sqrt{3}}\sin\sqrt{3}\,t\right]$

33. $f(t) = 5\,e^{4t} + 4t\,e^{4t} - 5\,e^{5t}$

34. $f(t) = -27(1+t)\,e^{-3t} + (28 - 8\,t)\,e^{-2t}$

35. $f(t) = -\frac{1}{16}\,e^{2t} + \frac{1}{2}\,t\,e^{2t} + \frac{1}{16}\,e^{-6t}$

36. $f(t) = \frac{8}{25}\,e^{-2t} + \frac{42}{25}\,e^{3t} + \frac{18}{5}\,t\,e^{3t}$

37. $f(t) = \frac{41}{7}e^{-3t} - \frac{13}{7}e^{-3t/2}\cos\frac{\sqrt{19}\,t}{5} - \frac{129\sqrt{19}}{133}e^{-3t/2}\sin\frac{\sqrt{19}\,t}{12}$

38. $f(t) = H_2(t)\sinh 2\,(t-2)$

39. $f(t) = H_1(t) + H_2(t) - H_3(t) - H_4(t)$

41. $(t) = \left(t\,e^{t/2}\right)/4$

42. $f(t) = \frac{1}{4}\,(t-1)\,e^{(t-1)/2}\,H_1(t)$

43. $f(t) = \frac{1}{2}\,e^{t/2}\,H_2(t/2)$

44. $f(t) = \frac{1}{4}\left(\frac{t}{4}\right)^3 e^{-3t/4}$

45. $f(t) = \frac{1}{6} e^{t/3}\left(e^{2t/3} - 1\right)$

46. $f(t) = \int_0^t 2 e^{-5p/2}\left[\sin(p/2)/p\right] dp$

49. $w = e^{-(x-2)^2}$

50. $F(s) = (3 e^{\pi s} - 1)\left[(s^2+1)(e^{\pi s} - 1)\right]^{-1}$

51. $y(t) = (2 + 3t)\, e^{-t} + 2 H_3(t)\left[(t-5) + (t-1) e^{-(t-3)}\right]$

52. $y(t) = 3\cos t - \sin t + \frac{1}{2} t \sin t + \frac{1}{2} H_{\pi/2}(t)\left[\left(t - \frac{\pi}{2}\right)\sin t - \cos t\right]$

53. $y(t) = t e^t + H_1(t)\left[2 + t + (2t-5) e^{t-1}\right] - H_2(t)\left[1 + t + (2t-7) e^{t-2}\right]$

54. $w = \frac{1}{5} + \frac{4 e^{-t}}{5}\cos 2t + \frac{2 e^{-t}}{5}\sin 2t + \frac{H_\pi(t)}{5}\left\{1 - e^{-(t-\pi)}\left[\cos 2(t-\pi) + \frac{\sin 2(t-\pi)}{2}\right]\right\}$

55. $R(x) = \frac{1}{2} - \frac{e^{-2x}}{2} - H_1(x)\left[1 - e^{-2(x-1)}\right] + H_2(x)\frac{1}{2}\left[1 - e^{-2(x-2)}\right]$

56. $y(t) = \cos t + H_1(t)\left[1 + (t-1) - \cos(t-1) - \sin(t-1)\right]$

57. $N(t) = 1 - e^{-t} - t e^{-t} + t^3 \frac{e^{-t}}{3} - H_2(t)\left\{1 - e^{-(t-2)} - (t-2) e^{-(t-2)} - \frac{(t-2)^2}{2} e^{-(t-2)} - \frac{(t-3)^3}{6} e^{-(t-2)}\right\}$

58. $y = e^{-t}\sin t + \frac{1}{2} H_\pi(t)\left[1 + e^{-(t-\pi)}\cos t + e^{-(t-\pi)}\sin t\right] - \frac{1}{2} H_{2\pi}(t)\left[1 - e^{-(t-2\pi)}\cos t - e^{-(t-2\pi)}\sin t\right]$

59. $y = \frac{1}{6}\left[1 - H_{2\pi}(t)\right](2\sin t - \sin 2t)$

60. $y = \frac{1}{6}(2\sin t - \sin 2t) - \frac{1}{6} H_\pi(t)(2\sin t + \sin 2t)$

61. $y = 1 + \sum_{n=1}^{\infty} (-1)^n H_{n\pi}(t)\left[1 - \cos(t - n\pi)\right]$

63. $z_1 = \frac{1}{18} - \frac{1}{10}\cos 2t - \frac{2}{45}\cos 3t - \left[\frac{1}{18} - \frac{1}{10}\cos(2t-2) - \frac{2}{45}\cos(3t-3)\right] H_1(t)$

$z_2 = \frac{2}{9} + \frac{4}{45}\cos 2t - \frac{14}{45}\cos 3t - \left[\frac{2}{9} + \frac{4}{45}\cos(2t-2) - \frac{14}{45}\cos(3t-3)\right] H_1(t)$

64. $I(t) = \frac{2}{\omega^2}(1 - \cos\omega t) + \frac{4}{\omega^2}\sum_{n=1} (-1)^n H_n(t)\left[1 - \cos\omega(t-n)\right]$

7.7 Προβλήματα

5. $y(t) = \left(\cosh\frac{t}{2} - 3\sinh\frac{t}{2}\right)e^{3t/2} - 2\,H_2(t)\sinh\frac{(t-2)}{2}\,e^{3(t-2)/2}$.

14. $i(t) = e^{-Rt/L}/L$, όχι **16.** $y(t) = \frac{1}{2}(\sin t - t\cos t) - H_\pi(t)\sin t$

17. $y(t) = \cos\left(\frac{\sqrt{3}}{2}t\right) + \sin\left(\frac{\sqrt{3}}{2}t\right) + \frac{8}{3}H_1(t)\sin\sqrt{\frac{3}{2}}(t-1) - \frac{4}{3}H_2(t)\sin\sqrt{\frac{3}{2}}(t-2)$

18. $y(t) = 3te^{-t} + \frac{1}{2}t^2e^{-t} + 3H_3(t)(t-3)e^{-(t-3)}$

19. $y = 2te^{-t} + H_{2\pi}(t)\left[1 - e^{-(t-2\pi)} - (t-2\pi)e^{-(t-2\pi)}\right]$

20. $y = \frac{\sqrt{2}}{2}e^{-t}\sin\sqrt{2}t + \frac{e^{-t}}{4}\cos\sqrt{2}t + \frac{1}{4}(\sin t - \cos t) + \frac{\sqrt{2}}{2}H_\pi(t)e^{-(t-\pi)}\sin\sqrt{2}(t-\pi)$

21. $y = \cos\omega t - \omega^{-1}H_{\pi/\omega}(t)\sin\omega t$ **22.** $y = [1 + H_\pi(t)]\sin t$

23. $y = 2H_1(t)e^{2(t-1)}(t-1) - H_2(t)e^{2(t-2)}(t-2)$

24. $N = \pi H_2(x)\sinh(x-2) - \pi H_4(x)\sinh(x-4)$

25. $w = 2\cos t - H_{\frac{3\pi}{2}}(t)\sin\left(t - \frac{3\pi}{2}\right)$ **26.** $R = \pi H_\pi(x)\sin 3(x-\pi) = -\pi H_\pi(x)\sin 3x$

27. $y(t) = \frac{P}{6EI}\left[3a - x^3 + (x-a)^3H_a(x)\right]$

7.8 Προβλήματα

1. $\frac{1}{16}[\sinh(2t) - \sin(2t)]$

2. $\cos(at)\left[\frac{\sin[(a-b)t]}{2(a-b)} + \frac{\sin[(a+b)t]}{2(a+b)}\right] + \sin(at)\left[\frac{\sin[(a-b)t]}{2(a-b)} - \frac{\sin[(a+b)t]}{2(a+b)}\right]$

3. $\frac{1}{a^2+b^2}[\cosh(bt) - \cos(at)]$ **4.** $\frac{\cosh(at) - \cosh(bt)}{a^2 - b^2}$

5. $\frac{1}{a^4}\left(1 + \frac{t}{2}\cos(at)\right) - \frac{3}{2a^5}\sin(at)$ **6.** $-\frac{1}{5}e^{-2t} + \frac{1}{3}e^{3t} + \frac{1}{6}e^{-3t}$

7. $\frac{1}{2}H_4 t - \frac{1}{2}H_4(t)e^{-2(t-4)}$ **8.** $\frac{5}{6}e^{5t} - \frac{5}{6}e^{3t}\cos(\sqrt{8}t) - \frac{5}{3\sqrt{8}}e^{3t}\sin(\sqrt{8}t)$

9. $\frac{1}{(n-1)!}t^{n-1}e^{at}$ **10.** $\frac{1}{16}\sin 2t - \frac{t}{8}\cos 2t$

11. $\frac{2}{3}e^{-2t} + \frac{1}{3}e^{t}$ **12.** $24\left[s^4(s^2+16)\right]^{-1}$

13. $(4 - s^2)\left[(s^2+1)(s^2+4)^2\right]^{-1}$

14. $\frac{\Gamma(3/2)}{s^{3/2}}\frac{d^3}{ds^3}\left(\frac{s}{s^2+1}\right) = \frac{-3\sqrt{\pi}\,(s^4 - 6s^2 + 1)}{s^{3/2}(s^2+1)^4}$

15. $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$

16. $\frac{a^2}{(s^2+a^2)^2}$

17. $\int_0^t f(a)\,da$

18. $\int_0^t \int_0^r f(a)\,da\,dr = t\int_0^t f(a)\,da - \int_0^t a\,f(a)\,da$

19. $\mathcal{L}^{-1}\left[\frac{F(s)}{s^2+a^2}\right]$

20. $\mathcal{L}^{-1}\left[\frac{F(s)}{s^2-a^2}\right]$

24. $\Gamma\left(\frac{4}{3}\right)\Gamma\left(\frac{8}{3}\right)\frac{t^3}{6}$

26. $\frac{\pi t^4}{64}$

28. (c) $I_{1/2}(p(t)) = \left(\sum_{j=0}^{n}\frac{a_j\, j!}{\Gamma(j+3/2)}\right) t^{j+1/2}$

29. (c) $I_v(p(t)) = \sum_{j=0}^{n}\left(a_j\, j!/\Gamma(j+v+1)\right)t^{j+v}$ (d) $I_v(e^{at}) = \sum_{n=0}^{\infty}\left(a_n/\Gamma(n+v+1)\right)t^{n+v}$

30. (i) Η $(D^{1/2}p)(t)$ δεν υπάρχει στο $t = 0$, αν $a_0 = 0$,

$$D^{1/2}p(t) = \sum_{j=0}^{n}\left(a_j\, j!/\Gamma(j+1/2)\right)t^{j-1/2}$$

31. $y = \frac{1}{42}f(t)*e^{3t} - \frac{1}{42}f(t)*e^{-3t} - \frac{\sqrt{2}}{28}f(t)*e^{\sqrt{2}t} - \frac{\sqrt{2}}{28}f(t)*e^{-\sqrt{2}t}$

32. $y = \frac{1}{15}f(t)*e^{3t} - \frac{1}{15}f(t)*\cos(\sqrt{6}t) - \frac{1}{5\sqrt{6}}f(t)*\sin(\sqrt{6}t)$

33. $y = \frac{1}{10}f(t)*e^{9t} - \frac{1}{10}f(t)*e^{-t} - \frac{1}{5}e^{9t} + \frac{1}{5}e^{-t}$

34. $y(t) = -\frac{e^{3t}*f(t)}{11} + \frac{e^{4t}*f(t)}{18} - \frac{7\cos(\sqrt{2}t)*f(t)}{198} - \frac{4\sqrt{2}}{18}f(t)*\sin(\sqrt{2}t)$

35. $y = -\frac{1}{2}f(t)*e^{-6t} + \frac{1}{2}f(t)*e^{-4t} + e^{-6t} - e^{-4t}$

36. $y = \int_0^t e^{-(t-r)}\sin(t-r)\sin ar\,dr$

37. $y = \frac{1}{2}\left(1 - e^{-t}\sin t - e^{-t}\cos t\right) - \frac{1}{2}H_{2\pi}(t)\left[1 - e^{-(t-2\pi)}\sin t - e^{-(t-2\pi)}\cos t\right]$

38. $y(t) = -\frac{1}{2}\sin 2t + \sin t$

39. $y(t) = 0.2\,e^{-t} + \cos 5t - 2\sin 5t$

40. $y(t) = \left(1 - e^{-2t}\right)/2$, για $0 < t < 1$ και $y(t) = e^{-t+1} - \frac{1}{2}\left(1 + e^2\right)e^{-2t}$, για $t > 1$

7.9 Προβλήματα

- Να λυθούν οι ακόλουθες ολοκληρωτικές εξισώσεις με χρήση μετασχηματισμού Laplace:

1. $y = \frac{1}{2}\,\delta(t) + \frac{3}{2}\sin t$ **2.** $y = \left(3\sqrt{\pi}\, t^{1/6}\right)\left(4\,\Gamma\left(\frac{4}{3}\right)\Gamma\left(\frac{7}{6}\right)\right)$

3. $y = \frac{2}{3}\,\delta'(t)$ (*υποδ.* $L\{\delta'\} = s$) **4.** $y = \pi^{-1}\,\delta'(t) + \pi$

5. $y = \left(t^{-1/2}/\pi\right)e^{-2t} + \sqrt{2/\pi}\; \mathrm{erf}\left(\sqrt{2t}\right)$ **6.** $y = \delta(t) + J'_0(t)$

7. $y = 2t - \frac{3}{2}\sin t$ **8.** $y(t) = -\frac{12}{25}e^{-t} + \frac{4}{5}t\,e^{-t} + \frac{12}{25}\cos 2t + \frac{9}{25}\sin 2t$

9. $y(t) = t$ **10.** $y = t^3 e^t/3 + t^2 e^{-t} + t\,e^{-t}$ **11.** $y(t) = \cos h\, t$

12. $y(t) = -\frac{e^{-t}}{2} + \frac{3}{2}\cos t + \frac{1}{2}\sin t$ **13.** $y(t) = \;\; 2\sin h\, t$

14. $(t) = -\frac{1}{8} + \frac{9}{4}t^2 + \frac{1}{8}\cos h\left(2\sqrt{3}\,t\right)$ **15.** $y(t) \equiv 0$

16. $y(t) = \frac{5}{4}e^t - \frac{1}{4}e^{-t} - \frac{1}{2}t\,e^{-t}$ **17.** $y(t) = e^{-t/2}\left(\cos\frac{\sqrt{15}}{2}t - \frac{1}{\sqrt{15}}\sin\frac{\sqrt{15}}{2}t\right)$

18. $y(t) = -\frac{1}{2}e^t + \frac{e^{-t}}{2} - t\,e^{-t}$ **19.** $y(t) = e^{-t}$ **20.** $y(t) = t + \frac{3}{2}\sin 2t$

21. $y(t) = 2 - 2\cos t$ **22.** $y(t) = 3$ **23.** $y(t) = \cos t + \sin t - 1$

24. $y(t) = \cos h\, t$ **25.** $y(t) = \frac{3}{8}e^{2t} + \frac{1}{8}e^{-2t} + \frac{1}{2}\cos 2t + \frac{1}{4}\sin 2t$

26. $y(t) = e^{-t/2}\cos\frac{\sqrt{3}\,t}{2} - \frac{1}{\sqrt{3}}e^{-t/2}\sin\frac{\sqrt{3}\,t}{2}$ **27.** $y(t) = \sin t - \frac{1}{2}t\sin t$

28. $i(t) = 20000\left[t\,e^{-100t} - (t-1)\,e^{-100(t-1)}H_1(t)\right]$

29. $i(t) = e^{-10t} - e^{-100t} + \left[\left(1 - e^{10}\right)e^{-10t} - \left(1 - e^{100}\right)e^{-100t}\right]H_1(t)$

30. $i(t) = \left[1 - H_{2\pi}(t)\right]\sin 100t$

31. $i(t) = 50^{-1}\left(1 - e^{-50t}\right)^2 - 50^{-1}H_1(t)\left\{1 + 98\,e^{-50(t-1)} - 99\,e^{-100(t-1)}\right\}$

32. $y(t) = \frac{e^{-3t}}{5} + \frac{4}{5}e^{2t}$ **33.** $y(t) = \frac{1}{2}t^2$

34. $y(t) = \frac{1}{2}\sin t + \left(1 - \frac{3}{2}t\right)e^{-t}$ **35.** $Y(s) = \sqrt{s^2+4}\left(s^2\sqrt{s^2+4} - 1\right)^{-1}$

36. $Y(s) = s\left(s^2 - 3s - \Gamma'(1) - \ln(s)\right)^{-1}$