

# ΑΠΑΝΤΗΣΕΙΣ ΠΡΟΒΛΗΜΑΤΩΝ

## ΚΕΦΑΛΑΙΟ 4

### 4.1 Προβλήματα

1.  $y = c_2 e^{c_1 x}$

2.  $A = c_1 e^{-1/4t} + c_2$

3.  $y = \tan(x/2)$

4.  $y = e^{2(x-1)}$

5.  $y = \frac{x^2}{2} + \frac{3}{2x^3}$

6.  $y = 2 + \ln \left| \frac{x^2 - 3}{2} \right|$  (μετασχηματισμός:  $z = (y')^{-1}$ )

7.  $x^5 - \frac{2}{x^3} + 2c_1 \frac{1}{3} + c_1 x + c_2, y \equiv c$

8.  $y = \pm \frac{1}{3} \left( \frac{2x + c}{1} \right)^{3/2} + c$

9.  $x = c_1 + c_2 y + y^3/6, y \equiv c$

10.  $y = c_1 \tan\left(c_2 - c_1 \frac{x}{2}\right)$  ή  $y = c_1 \frac{c_2 e^{c_1 x} - c_2 e^{-c_1 x}}{c_2 e^{c_1 x} + 1}$  ή  $y = 2(x + c)^{-1}$

11.  $y = c_1 + c_2 x^3 + \frac{x^4}{4} - \frac{3x^5}{10} + \frac{x^6}{18}$

12.  $y - \ln|e^y + c_1| = c_1 x + c_2$ , για  $c_1 \neq 0$  ή  $y = -\ln(c - x)$  για  $x < c$  ή  $y \equiv c$

13.  $x = c_2 - c_1 y + (1 + c_1^2) \ln|y + c_1|$  ή  $y \equiv c$

14.  $y = \pm (2/3)(x - 2c_1) \sqrt{x + c_1} + c_2$  ή  $y \equiv c$  15.  $x + c_2 = \pm (2/3)(y - 2c_1)(y + c_1)^{1/2}$

16.  $y \ln|y| - y + c_1 y + x = c_2$ , ή  $y = c$

17.  $e^y = (x + c_2)^2 + c_1$

23.  $\frac{a}{2} (e^{x/a} + e^{-x/a}) = a \operatorname{ch} \left(\frac{x}{a}\right)$ : αλυσοειδής καμπύλη

### 4.2 Προβλήματα

10.  $3x^2 - 8x + 2$

11. 0

12.  $(r^2 - 4r + 3) e^{rx}$

16. όχι

17. όχι

18. ναι,  $(-\infty, 1)$

19. ναι,  $\mathbb{R}$

20. ναι,  $(0, \infty)$

21. ναι,  $(0, 3)$

22. ναι,  $(0, 5)$

23. ναι,  $(-\infty, +\infty)$

24. (a) 2, (b)  $x \sin x + \cos x$

25. (a)  $3x^2 - x^3$ , (b)  $6x + 3x^2 - 2x^3$ , (c)  $3x^2 + 2x^3$ , (d)  $6x + 3x^2 - 2x^3$

(e)  $D^2 + D - 2$ , (f)  $6x + 3x^2 - 2x^3$

**4.3 Προβλήματα**

1.  $-(b \sin at \sin bt + a \cos at \cos bt)$       2. 0
3.  $(b - a)e^{(a+b)t}$       4.  $-be^{2at}$       5. t
6.  $W(1) = 5e^3$ ,  $W(-1) = 5e^{-3}$ ,  $W(3) = 5e^{-9}$ ,  $W(x) = 5e^{3x}$
8.  $y'' + y' - 6y = 0$       9.  $x^2 y'' - 3xy' + 4y = 0$
10.  $xy'' - y' + 4x^3 y = 0$       11.  $y'' + (y')^2 + 1 = 0$
12.  $yy'' + (y')^2 = 2$       13.  $yy'' + (y')^3 = 0$
14. ναι      15. όχι      17. ναι      18. ναι      19. ναι      20. ναι
22.  $J = (-2, 2)$ .      23.  $J = (0, +\infty)$ .
31.  $y = c_1 e^{2x} + c_2 x e^{2x}$       32.  $y = c_1 \sqrt{x} + c_2 \sqrt{x} \ln|x|$
33.  $y = c_1 e^{2x} + c_2 (2x^2 + 2x + 1)$       34.  $y = c_1 x \cos x + c_2 x \sin x$
35.  $y = c_1 x^2 + c_2 (2x + 1)$       36.  $y = c_1 x^3 + c_2 x^{-3}$
37.  $y = c_1 x^2 \sin x + c_2 x^2 \cos x$       38.  $y = c_1 x^{3/2} + c_2 x^{3/2} \ln|x|$
39.  $y = c_1 x^{-2} + c_2 x^{-3}$       41.  $y(x) = (x^2 + 1)[c_1 + c_2 (x(x^2 + 1)^{-1} + \tan^{-1} x)]$
42.  $y(x) = c_1 \frac{x}{(x+1)^2} + c_2 \frac{x^2 - 2x \ln x - 1}{(x+1)^2}$

**4.4 Προβλήματα**

1.  $y = c_1 e^{x/2} + c_2 e^{4x/3}$       2.  $y = c_1 e^{(-1+\sqrt{2})x} + c_2 e^{(-1-\sqrt{2})x}$
3.  $y = c_1 e^{3x} + c_2 e^{(-2+\sqrt{2})x} + c_3 e^{(-2-\sqrt{2})x}$       4.  $y = c_1 + c_2 e^x + c_3 e^{2x} + c_4 e^{-2x}$
5.  $y = c_1 e^{\sqrt{a}x} + c_2 e^{-\sqrt{a}x} + c_3 \cos \sqrt{a}x + c_4 \sin \sqrt{a}x$       6.  $y = c_1 e^{-6kx} + c_2 e^{2kx}$
7.  $y = c_1 + c_2 x + c_3 x^2 + c_4 x^3$       8.  $y = (c_1 + c_2 x + c_3 x^2) e^{2x}$
9.  $y = c_1 + c_2 x + c_3 x^2 + c_4 e^{-3x}$       10.  $y = (c_1 + c_2 x) e^{2x} + (c_3 + c_4 x) e^{-3x}$
11.  $y = c_1 + c_2 x + c_3 \cos 2x + c_4 \sin 2x$       12.  $y = 2e^{4x} + e^{-3x}$
13.  $y = (3x + 2) e^{-3x}$       14.  $y = (13x + 3) e^{-2x}$

15.  $y = e^{2x} \sin(5x)$

16.  $y = e^{-3x}(4 \sin 2x + 3 \cos 2x)$

17.  $y = 3e^{-x/3} \left( \sin\left(\frac{2x}{3}\right) + 2 \cos\left(\frac{2x}{3}\right) \right)$

18.  $y = e^x - 2e^{2x} + e^{3x}$

19.  $y = (32e^{-x} - 23e^{2x} + 6xe^{2x}) / 9$

20.  $y = [9e^{x/3} + (4x - 9)e^{-x}] / 16$

21.  $y = \frac{1}{4} e^{2x-\pi} \sin(4x)$

22.  $y = \sin\sqrt{2}(x + \sqrt{\pi})$

23.  $B = e^s + e^{-s} + \cos s - \sin s$

24.  $y = \cos\sqrt{3}(x - 3) - \frac{1}{\sqrt{3}} \sin\sqrt{3}(x - 3)$

**4.5 Προβλήματα**

2.  $y = c_1 e^{3x} + c_2 e^{-x} - 4xe^{-x}(2x+1)$      3.  $y = \left( c_1 + c_2 x + \frac{1}{2} x^2 \ln x - \frac{3}{4} x^2 \right) e^{-x}$

4.  $y = \left\{ c_1 \cos 2x + c_2 \sin 2x + \frac{1}{2} x \sin 2x + \frac{1}{4} \cos 2x \ln |\cos 2x| \right\} e^x$

5.  $y = [c_1 + \ln(1 + e^{-x})] e^x + [c_2 + \ln(1 + e^{-x})] e^{2x}$

6.  $u(t) = c_1 e^t + c_2 e^{-t} + c_3 e^{-2t} + \frac{1}{6} \int_{t_0}^t [2e^{-2(t-s)} - 3e^{-(t-s)} + e^{t-s}] f(s) ds$

7.  $u(t) = (2 - 3c)e^{-t} + ce^t + (2c - 1)e^{-2t} + \frac{1}{6} \int_0^t [2e^{-2(t-s)} - 3e^{-(t-s)} + e^{t-s}] f(s) ds$ ,  $c$  σταθερά

8.  $u(t) = c_1(t^{-1} - 1) + c_2(t - 1) + \frac{1}{2} \int_1^t \left( \frac{1}{2} t^{-1} + \frac{1}{2} ts^{-2} - 2s^{-1} \right) f(s) ds$

9.  $R(x) = (c_1 + c_2 x - \ln|x|) e^{-3x}$ ,  $x < 0$

10.  $y(x) = c_1 \cos x + c_2 \sin x + \tan x \sin x - \frac{1}{2} \tan^2 x \cos x$

11.  $y(x) = c_1 x + c_2 x^2 - x \ln x - \frac{1}{2} x(\ln x)^2$

12.  $N(x) = c_1 + c_2 \cos x + c_3 \sin x + \ln|\sec x + \tan x| - x \cos x + \sin x \ln|\cos x|$

13.  $y = e^x \left( c_1 \cos \sqrt{3} x + c_2 \sin \sqrt{3} x + \frac{\sin x}{2} \right)$

14.  $y(x) = c_1 e^{2x} + c_2 e^{3x} + e^x [(Ax+B)\cos 2x + (Cx+D)\sin 2x]$

15.  $y(x) = c_1 \cos x + c_2 \sin x + x[(A_0 + A_1 x + A_2 x^2)\cos x + (B_0 + B_1 x + B_2 x^2)\sin x]$

16.  $y = 2 \cos 2t - \frac{2}{3} \sin 2t + \frac{1}{3} \sin t$      17.  $y = \left( \frac{46}{50} - \frac{135x}{50} + \frac{x^2}{2} \right) e^{3x} + \frac{4}{50} \cos x - \frac{3}{50} \sin x$

$$18. y(s) = -2e^{-s} + \frac{5}{6}e^{-2s} - \frac{1}{3}e^s + \frac{3}{2}$$

$$19. y(t) = 2e^t - e^{-t} - 2e^{-t}\sin t$$

$$20. y(t) = 8e^{2t} - 24e^t + 4t^2 + 12t + 14 + 2e^{-t}$$

$$21. y = c_1 \cos 2t + c_2 \sin 2t + \left(\frac{1}{4}t^2 - \frac{1}{8}\right) + \frac{3}{64}(8t^2 \sin 2t - 3 \sin 2t + 4t \cos 2t)$$

$$22. y(t) = (e^{-t} + 7e^t \cos 2t + e^t \sin 2t) / 8$$

$$23. y(t) = -\frac{73}{300}e^{2t} - \frac{51}{50}te^{2t} + \frac{1}{3}e^{-t} + t + \frac{1}{2}t^2 + \frac{3}{25}\sin t + \frac{4}{25}\cos t + \frac{3}{4}$$

$$24. y(t) = \frac{31}{128}e^{2t} + \frac{97}{128}e^{-2t} + \frac{1}{32}te^{2t} - \frac{1}{16}t^2e^{2t} + \frac{1}{12}t^3e^{2t}$$

$$27. y(t) = c_1 \cos t + c_2 \sin t + z(t), \text{ όπου } z(t) = t, \text{ για } t \in [0, \pi], z(t) = -\frac{\pi}{2}t \sin t, \text{ για } t \in (\pi, 2\pi) \text{ και } z(t) \equiv 0, \text{ για } t \geq 2\pi.$$

$$29. \varphi(t) = (c_1 - t) \cos(kt) + (c_2 - t) \sin(kt)$$

$$30. y(x) = c_1 \cos(\lambda x) + c_2 \sin(\lambda x) + \sum_{n=1}^N \frac{k_n}{\lambda^2 - n^2 p^2} \sin(n\pi x)$$

$$33. 3x^2.$$

$$34. (i) y(x) = \frac{1}{10}(\cos x - 3 \sin x) \quad (ii) y(x) = \frac{1}{10}(\sin x + 3 \cos x).$$

#### 4.6 Προβλήματα

$$1. y = \frac{1}{2}x^5 e^{-2x}$$

$$2. y = \frac{1}{2}e^{5x}$$

$$3. y = 2x^3 + 9x^2 + 40x + 73$$

$$4. y = -\frac{1}{60}x^5 - \frac{1}{3}x^3 - 2x$$

$$5. y = -x^{10} - 151,2x^4$$

$$6. y = x^4 + 4x^3 + 24x^2 + 69x + 117$$

$$7. y = -2x^3 - 5x^2 - 10x - 10$$

$$8. y = -e^{2x}(x^3 + 6x)$$

$$9. y = \frac{1}{8}e^{2x}(4x^3 - 2x^2 - 18x - 25)$$

$$10. y = 2e^{-2x}(x^2 + 4x + 6) + \frac{1}{3}e^{2x}$$

$$11. y = x^3 - 1$$

$$12. y = \frac{1}{2}x(\ln x - 1)$$

$$13. y = \frac{1}{48}(4x^3 - 6x^2 + 6x - 3)$$

**4.7 Προβλήματα**

$$4. y_p = \int_0^t K(t, s) f(s) ds = \sum_{j=0}^{n-1} \int_{2j\pi}^{(2j+1)\pi} K(t, s) ds + \int_{2n\pi}^t K(t, s) ds ,$$

όπου  $K(t, s) = e^{(t-s)/10} \sin(t-s)$  και  $2n\pi \leq t \leq (2n+1)\pi$ .

$$9. 2\pi \sqrt{\frac{1,99}{50,5(32,2)}} \leq T \leq 2\pi \sqrt{\frac{2,01}{49,5(32,2)}}.$$

$$10. q(t) = -\frac{9}{26} e^{-t} \cos 3t - \frac{7}{26} e^{-t} \sin 3t + \frac{1}{26} (9 \cos 2t + 6 \sin 2t)$$

$$\frac{4}{51} \cos 20t - \frac{1}{51} \sin 20t, \text{ συχνότητα συντονισμού } 5/\pi.$$

$$12. i_p = \frac{4}{51} \cos 20t - \frac{1}{51} \sin 20t, \text{ συχνότητα συντονισμού } 5/\pi.$$

$$13. q = -\frac{5}{12} \sin 3t + \frac{5}{4} \sin t, \quad i = -\frac{5}{4} \cos 3t + \frac{5}{4} \cos t, \quad i_0 = 0.$$

**4.8 ΑΝΑΚΕΦΑΛΑΙΩΣΗ – ΠΡΟΒΛΗΜΑΤΑ ΕΠΙΣΚΟΠΗΣΗΣ**

$$3. -\frac{m}{2} \leq x \leq 0, \quad -\frac{m}{3t} \leq x' \leq \frac{m}{3t}$$

$$7. y(t) = (1 + \ln t)/4$$

