ΑΠΑΝΤΗΣΕΙΣ ΠΡΟΒΛΗΜΑΤΩΝ

ΚΕΦΑΛΑΙΟ 2

2.1

1. ναι

2. ναι

3. ναι

4. ναι

5. όχι

6. όχι

7. $y(t)=\tan\left(t-(1/2)t^2+c\right)$

8. $y(t)=\arcsin(c\sin t)$

9. $y^2+c\,(y^2\sin x+1)=0$

10. $y=\ln|\cos(c-x)|+k$

11. $y=c(1-c\cos x)^{-1}$

12. $\ln|1+y|=x+x^2/2+c$

13. $\dfrac{1}{3y^3}-\dfrac{2}{y}=\dfrac{1}{x}+\ln|x|+c$

14. $y=(cx-3)(2x-1)^{-1}$

15. $y=c+c_1[(x-a)/(x-b)]^{\frac{1}{a-b}}$

16. γενική $a^2-x^2=c\cos^2 y$ με $x^2\neq a^2$, $y\neq(\pi/2)+n\pi$,
ιδιάζουσα: $y=(\pi/2)+n\pi$

17. γενική $e^{x+1}+\ln(\csc y-\cot y)+\cos y=c$, $y\neq n\pi$, ιδιάζουσα: $y=n\pi$

18. γενική $\ln|x|\left(y+\sqrt{1+y^2}\right)=c$, $x\neq 1$, $x\neq 0$, ιδιάζουσες: $x=1$, $x=0$

19. γενική $\sin y=(x-1)^2e^{2x+c}$, $x\neq 1$, $y\neq n\pi$, ιδιάζουσα: $y=n\pi$

20. γενική $x^2+2x=e^{-y^2}+c$, $x\neq -1$, ιδιάζουσα: $x=-1$

21. $y(t)=\left[9+2\ln\left(\dfrac{1+t^2}{5}\right)\right]^{1/2}$, $-\infty<t<\infty$

22. $y(t)=1-(4+2t+2t^2+t^3)^{1/2}$, $-2<t<\infty$

23. $a=b$ $\quad y(t)=\dfrac{a^2kt}{1+akt}$, $\quad \dfrac{-1}{ak}<t<\infty$,

$a\neq b$ $\quad y(t)=\dfrac{ab\left[1-e^{k(b-a)t}\right]}{a-be^{k(b-a)t}}$, $\quad \dfrac{\ln(a/b)}{k(b-a)}<t<\infty$

24. $y(t)=2e^t/(2e^t-1)$, $t\in\mathbb{R}$

25. $y=a\left(1+\sqrt{1-x^2}\right)$

26. $y \equiv 0$ **27.** $\cos^2 x \cos 2y = -1$

28. $3y^2 = 1 + 2e^{3x^2 - 12}$

29. $y = 1 - \sqrt{x^3 + 2x^2 + 2x + 4}$, $x \in [-2, +\infty)$ **30.** $\ln|y| + y^2 = \sin x + 1$, $y > 0$,

32. $y(x) = \begin{cases} (1 - x^2)^2, & \text{για } x \le -1, \\ 2(1 - x^2)^2, & \text{για } -1 \le x \le 1, \\ 0, & \text{για } 1 \le x. \end{cases}$

34. $y(x) = \begin{cases} c_1 x^3, & x \ne 0, \\ c_2 x^3, & x \ne 0. \end{cases}$ Για $c_1 = c_2 = 1$ η $y(x) = x^3$, $x \in$ IR, ειδική λύση

36. a. $y = 1 - 2x - \ln|c - x|$ b. $y = 1 + x + (1 + 2ce^{-3x})/(1 - ce^{-3x})$

c. $2y - 2x + \sin 2(x + y) = c$ d. $4(y - 2x + 3) = (x + c)^2$

38. d. $y^3 - 4y - x^3 = -1$, $|x^3 - 1| < \dfrac{16}{3\sqrt{3}}$ ή $-1{,}28 < x < 1{,}6$

42. a. $y = \dfrac{a}{g}x + \dfrac{bg - ad}{g^2}\ln|gx + d| + c$, $g \ne 0$, $gx + d \ne 0$

b. $x = \dfrac{g}{a}y + \dfrac{ad - bg}{a^2}\ln|ay + b| + c$, $a \ne 0$, $ay + b \ne 0$

43. $y' = -\dfrac{x}{2y} \Rightarrow x^2 + 2y^2 = k$ **44.** $y' = \dfrac{4y}{x} \Rightarrow y = kx^4$

45. $x\left[(x - k)^2 + y^2 - k^2 + 1\right] = 0$, $k^2 > 1$ **46.** $y' = \dfrac{x}{y} \Rightarrow x^2 - y^2 = k$

47. $x - y = k(x + y)^3$ **48.** $x^2 - 2ky = k^2$, $k > 0$

2.2

1. $e^x + (x^2/2) + (y^2/2)$ **2.** όχι πλήρης

3. $x^3y - xy^3 + (y^2/2) = c$ **4.** $t^2 + z^2 = c$

5. $e^y - 2x^2y - \log|x| = c$ **6.** όχι πλήρης

7. όχι πλήρης **8.** $x^2y - \tan x + y^2 = c$

9. $y = (c - x)(2 + xe^x)^{-1}$ **10.** $y = (c - 3x)(x^2 - 1)^{-1}$

11. όχι πλήρης **12.** $\sin x \cos y + x^2 - y^2 = c$

13. $r = (c - e^{\theta})\sec\theta$

14. $y = [c + e^t(t-1)](1+e^t)^{-1}$

15. $x^2 + x\,y^2 - \sin(x+y) - e^y = c$

16. όχι πλήρης

17. $\cos(xy^{-1}) = c$ ή $xy^{-1} = c$

18. όχι πλήρης

19. $\ln\left|\dfrac{1+xy}{1-xy}\right| - 2x = c$

20. $x\,e^{y^2} + \csc y \cot x = c$

21. $x^2 + y^2 = c$

22. $x^3(1+\ln y) - y^2 = c$

23. $3x^2y + y^3 = 32$

24. $x^2 + 2xy - y^2 = -16$

25. $x\sin y + y^2 = \pi^2$

26. $x^2e^{2y} + 2y = 4$

27. $y = [x + \sqrt{28 - 3x^2}]/2$, $|x| < \sqrt{28/3}$

28. $y = [x - (24x^3 + x^2 - 8x - 16)^{1/2}]/4$

29. $xe^x y^3 + e^x - 6y^3 = -5$

30. $2\cos x\cos y = 1$

31. $e^{xy^2} + x^4 - y^3 = 2$

32. $y = (e-x)(e^x - 1)^{-1}$

33. $y = -2(te^t + 2)^{-1}$

34. a. $N(x,y) = x\cos(xy) + g(y)$ b. $N(x,y) = x\,e^{xy} - x^4 + g(y)$ όπου g αυθαίρετη συνάρτηση του y.

36. a. $k = 3$, $x^2y^2 + 2x^3y = c$ b. $k = 1$, $x^2 + e^{2xy} = c$

37. $2x = y^3 + cy$

38. $xy - x - 2y = c$

39. $x^3y + e^y = c$

40. $x + \arctan(yx^{-1}) = c$

2.3

1. $\mu = y^{-4}$, $x^2y^{-3} - y^{-1} = c$, $y \equiv 0$

2. $\mu = x^{-1}$, $xy - \ln x - 2^{-1}y^2 = c$, $x \equiv 0$

3. $\mu = \dfrac{1}{x^3y^3}$, $-\dfrac{1}{2x^2y^2} + \dfrac{3}{2}y^2 = c$, $x \equiv 0$, $y \equiv 0$

4. $\mu(t) = e^t$, $y(t) = -e^t \pm (e^{2t} + 2ce^{-t})^{1/2}$

5. $\mu(t) = e^{-at}\cos y\,(a = ;)$, $y(t) = \arcsin[(c-t)e^t]$

6. $\mu = ty$, $\ln\left|\dfrac{y}{t}\right| + ty = c$, $t \equiv 0$, $y \equiv 0$

7. $\mu = t^{-2}y^{-3}$, $\dfrac{t^2}{y^2} + \dfrac{1}{t} = c$, $t \equiv 0$, $y \equiv 0$

8. $\mu = -t^{-1}y^{-2}$, $2\ln|ty| + \frac{t}{y} = c, t \equiv 0, y \equiv 0$ **9.** $\mu = xy$, $x^3y + 3x^2 + y^3 = c$

10. $\mu = (x^2 - y^2)^{-1}$, $x - y + \ln\sqrt{x+y} - \ln\sqrt{x-y} = c$, $y = x$, $y = -x$

11. $\mu = e^{-y}$, $2xe^{x-y} + y^2 = c$ **12.** $\mu = (xy)^{-4/3}$, $(y - x + 1)^3 = c\,xy$, $x \equiv 0$, $y \equiv 0$

13. $\mu = (xy(x+y+1))^{-1}$, $(x+y+1)^3 = c\,xy$, $x \equiv 0$, $y \equiv 0$

14. $\mu = x$, $6x^2y^2 + 8x^3y + 3x^4 = c$, $x \equiv 0$

15. $\mu = (x^2 + y^2)^{1}$, $y + \arctan\left(\frac{y}{x}\right) = c$, $y \equiv 0$

16. $\mu = x^a y^b$ $(a = ;\ b = ;)$, $x^7y^2 - x^3y^9 = c$, $x \equiv 0,\ y \equiv 0$

17. $\mu = e^{ax}e^{by}$ $(a = ;\ b = ;)$, $(x^2 - 2xy)e^{-x+2y} = c$, $x \equiv 0$

18. $\mu = x^a e^{bx}$ $(a = ;\ b = ;)$, $\frac{x^2}{2}ye^{6x} + \frac{e^{6x}}{9}(6x - 1) = c$, $x \equiv 0$

19. $\mu = x^a y^b$ $(a = ;\ b = ;)$, $x^6y^4 + x^5y^7 = c$, $x \equiv 0, y \equiv 0$

20. $\mu = x^a y^b$ $(a = ;\ b = ;)$, $x^3y^4 = 1296$

21. $\mu = y^a e^{x^b}$ $(a = ;\ b = ;)$, $y^3e^{x^2} = 125\,e^9$

22. $\mu = x^a\ y^b\ e^{x^g}$ $(a = ;\ b = ;\ g = ;)$, $yx^2\,e^{x^2} = 12\,e^4$

23. $\mu = x$, $x^2y^3 - 2x^2 = -1143$

24. $\mu = e^{x^b}$ $(b = ;)$, $xy\,e^{x^2} = -2e$ **25.** μ = σταθερά, $4x^2 + 3y^2 = 247$

26. $\mu = x^a e^b$ $(a = ;\ b = ;)$, $3x^{2/3}ye^x = -21e$

2.4

1. $y = e^{-x}\arctan^{-1}(e^x) + ce^{-x}$

2. $y = (1 + x^2)^{-1}\ln(\sin x) + c(1 + x^2)^{-1}$

3. $y = x^2e^{-x} + x^2 - 2x + 2 + ce^{-x}$

4. $y = x^2\csc x + c\csc x$

5. $xy\sin x = \sin x - x\cos x + c$

6. $y = (x^3 + c)/\ln x$

7. $y = \frac{c}{x} + \frac{3\cos 2x}{4x} + \frac{3}{2}\sin 2x$

8. $y = (\arctan x + c)(1 + x^2)^{-2}$

9. $y = 1 - \frac{1}{2} e^{-2x}$, $-\infty < x < \infty$ **10.** $y = \frac{q}{p} + \left(y_0 - \frac{q}{p}\right) e^{-p(x-x_0)}$, $x \in \mathbb{R}$

11. $y = \frac{x}{2} - \frac{5}{2x}$, $0 < x < \infty$

12. $y = (x + x_0^2 y_0 - x_0) x^2$, $0 < x < \infty$, αν $x_0 > 0$ και $-\infty < x < 0$ αν $x_0 < 0$

13. $y = x + \ln\left((x/x_0) + x_0 y_0 - x_0^2\right) / x$, $0 < x < \infty$, αν $x_0 > 0$ και $-\infty < x < 0$, αν $x_0 < 0$.

14. $y = [a + \ln(1+x)]e^{-x}$, $-1 < x < \infty$

15. $y = \frac{1}{4}x^2 - \frac{1}{3}x + \frac{1}{2} + \frac{1}{12}x^{-2}$, $0 < x < \infty$

16. $y = \tan x + a \sec x$, $-\pi/2 < x < \pi/2$

17. $y(x) \to 0$, όταν $\lambda < 0$, $y(x) \equiv y_0$, όταν $\lambda = 0$ και $y(x) \to +\infty$, όταν $\lambda > 0$.

19. $y(x) = x^2$, όταν $x \le 0$ και $y(x) = x^2 - 2x^3$, όταν $x \ge 0$.

20. $y = \frac{(x-1)^3}{6} + c_1 \ln|x-1| + c_2$ **21.** $y(x) = \begin{cases} e^x, & x < 0 \\ 2e^x - x - 1, & x \ge 0 \end{cases}$

22. $y(x) = \begin{cases} e^{-x}(x+1), & 0 \le x < 2 \\ 2e^{-x} + e^{-2}, & \end{cases}$ **23.** $y(x) = \begin{cases} \frac{1}{2}\left(1 - e^{-2x}\right), & 0 \le x \le 1 \\ \frac{1}{2}\left(e^2 - 1\right)e^{-2x}, & 1 < x \end{cases}$

24. $y(x) = \begin{cases} e^{-2x}, & 0 \le x \le 1 \\ e^{-(x+1)}, & 1 < x \end{cases}$ **27.** ναι, αν $y \neq 0$ **28.** ναι **29.** ναι

30. όχι **31.** ναι **32.** ναι **33.** ναι **34.** ναι **35.** $y = x^2 \ln|x| + cx^2$

36. $y = \sin x + c\cos x$ **37.** $y = \frac{1}{2} - \frac{1}{x} + \frac{c}{x^2}$

38. $y = \frac{x^2(x-1)}{2} + c(1-x)$ **39.** $y = e^{-x} + \frac{ce^{-x}}{x}$

40. $3y^4x^8 - 2x^6 = c$ **41.** $x^2y^{-2} + 2e^x = c$

42. $-3x^2y^{-2/3} - 4x = c$ **43.** $u = xy^{-2}$, $y = \frac{2}{3 + cx^2}$

44. $y = \pm\left[k/(1 + cke^{-kx})\right]^{1/2}$

45. $y = \pm\sigma^{1/2}(x)\left[2\int^x \sigma(s)ds + c\right]^{-1/2}$, όπου $\sigma(x) = \exp(2\Gamma\sin x + 2Tx)$

2.5

1. $y = x + (c - x)^{-1}$

2. $y = x^{-1} + 2x\,(c - x^2)^{-1}$

3. $y = \sin x + (c\cos x - (1/2)\sin x)$

4. $y = (x - 2 + ce^{-x})^{-1} + 1$

5. $y = (2 + ce^{-2x^2})^{-1} + x$

6. $y = 1 + (1 - t + ce^{-t})^{-1}$

7. $y = e^t + (ce^{-3t} - e^{-t}/2)^{-1}$

8. $y = (1 - tce^{-t})(1 - ce^{-t})^{-1}$

9. $y = t + t\left(c - \dfrac{t^5}{5}\right)^{-1}$

10. $y = \dfrac{2}{x} + \dfrac{1}{-x^4}\,/\,cx^{-3}$

11. $y = -e^x + (ce^{-x} - 1)^{-1}$

12. $y = -x^2 + \left(2x^2 e^{x^2}\right)\left(e^{x^2} + c\right)^{-1}$

13. $y = \dfrac{1}{\cos x} + \dfrac{3\cos^2 x}{c - \cos^3 x}$

14. $y = \dfrac{1}{x} + 2(cx^3 - x)^{-1}$

15. $y = x + 2x\left(cx^2 + 1\right)^{-1}$

16. $y = x + (ce^{-x} - 1)^{-1}$

17. $y = 1 + x(1 - x + ce^{-x})$

23. $p^{-1}\int (my^2 + ny + p)^{-1}\,dx = C - \int f_0(x)\,dx$

25. Συνθήκη: $(b' - 2f_1 b)f_2^{-1}\,b^{-2} = -\left(k + \dfrac{1}{k}\right)$

2.6

1. βαθμού 3

2. βαθμού 2

3. όχι

4. βαθμού 0

5. βαθμού -2

6. βαθμού $-\dfrac{1}{3}$

7. όχι

8. βαθμού 3

11. $x + y\ln|x| = cy$

12. $\ln(x^2 + y^2) + 2\tan^{-1}\left(\dfrac{y}{x}\right) = c$

13. $4x = y(\ln|y| - c)^2$

14. $y^9 = c(x^3 + y^3)^2$

15. $\left(\dfrac{y}{x}\right)^2 = 2\ln|x| + c$

16. $x\cos\left(\dfrac{y}{x}\right) = c$

17. $y + x = cx^2 e^{y/x}$

18. $y^2 - cx = y\sqrt{y^2 - x^2}$

19. $y\sin\dfrac{y}{x} = c$

20. $y = c\left(1 + \ln\dfrac{y}{x}\right)$

21. $2e^{x/y} + \ln y = c$

22. $\ln x^2 - e^{-y/x}\left(\sin\frac{y}{x} + \cos\frac{y}{x}\right) = c$

23. $y = 2x\tan\left(2\ln|x| + \tan^{-1}(-0.5)\right)$

24. $y^3 + 3x^3\ln|x| = 8x^3$

25. $\ln|x| = e^{y/x} - 1$

26. $4x\ln\left|\frac{y}{x}\right| + x\ln x + y - x = 0$

27. $3x^{3/2}\ln x + 3x^{1/2}y + 2y^{3/2} = 5x^{3/2}$

28. $(x+y)\ln|y| + x = 0$

29. $\ln|y| = -2(1 - x/y)^{1/2} + \sqrt{2}$

30. $y^2\ln x = 2y^2 + xy - x^2$

32. $x - 4 = c\left(1 - \frac{4(y+1)}{x-4}\right)^{-1/4}$

33. $3(x-2)^2 - 2(x-2)(y+3) - (y+3)^2 = c$

34. $-8\ln|x - 2y + 4| + 2x - 6y = c$

35. $-\frac{2}{\sqrt{15}}\tan^{-1}\left(\frac{2(y-1) - 3\sqrt{5}}{\sqrt{5}(x+1)}\right) = \ln|x| + c$

36. $\frac{5}{16}(-2u + z) + \frac{13}{128}\ln|16u - 8z| = -\frac{1}{2}u + c$

37. $\left[5\left(\frac{x-3}{t-4}\right)^2 - 2\left(\frac{x-3}{t-4}\right) + 2\right]^{-1/2} = c(t-4)$

38. (i) $(x + y - 3)^3 = c(2x + y - 4)^2$

(ii) $x + 2y + c = 3\ln|x + y + 2|$

(iii) $(x - y - 4)^3 = c(x + y - 2)$

(iv) $(2y - x + 3)^2 = c(y - x + 2)$

2.7

1. $c^2 + cx^2 = 2y$, $8y = -x^4$

2. $12y = c(c + 4x^3)$, $3y = -x^6$

3. $2c^3x^3 = 1 - 6c^2y$, $2y = x^2$

4. $x^3(y + c^2) + c = 0$, $4x^6y = 1$

5. $x^2 = c(y - c)$, $y = \pm 2x$

6. $xy = c(3cx - 1)$, $12x^2y = -1$

7. $xc^2 + (x - y)c + 1 - y = 0$, $(x + y)^2 = 4x$

8. $3xy = c(xc^2 - 3)$, $9x^3y^2 = 4$

9. $2c^3y = c^4x^2 + 12$, $3y^2 = \pm 8x^3$

10. $p^3(x + 2p)^2 = c$, $y = 3xp + 5p^2$

11. $x^2 = cp^{-4/3} - 2p^{-1}$, $y = xp + x^3p^2$

12. $y = cx + 1 - \ln c$, $y = 2 + \tan x$

13. $y = cx - c^3$, $27y^2 = 4x^3$

14. $y = cx - e^c$, $y = x\ln x - x$

15. $y = cx + c^2 + 1$, $y = 1 - x^2/4$

16. $y = cx - c^{2/3}$, $27x^2y + 4 = 0$

17. $y = cx + \sqrt{1 + c^2}$, $y = \sqrt{1 - x^2}$, $y > 0$

18. $y^3 = cx + 2c^2/3$

19. $(27ay^2-16x^3)y^2+16a^2x(9ay^2-4x^3)c-128a^3x^2c^2-64a^4c^3=0$

20. $y^2(4y-3x^2)+6x(2x^2-3y)c+9c^2=0$

2.8

1. $\sin y=x^2-2x+2+ce^{-x}$

2. $\sqrt{y+1}=x+1+c\sqrt{x+1}$

3. $\cos y=1-ce^{-\cos x}$

4. $y=c+\sqrt{x^2+y^2}$

5. $e^{-2x}=2y^2(\ln|y|-c)$, $y\neq 0$, $y=y(x)\equiv 0$

6. $e^{xy^2}+x^4-y^3=c$

7. $3(x^2+2y-1)^{1/3}=2x+c$

8. $x^2e^{2y}=2x\ln x-2x+c$

9. $-e^{y/x^4}=x^2+c$

10. $\ln(\tan y)=x+cx^{-1}$

11. $x^3y^3=2x^3-9\ln|x|+c$

12. $e^y=-e^{-x}\cos x+ce^{-x}$

2.9

1. $y^2+yx=cx^3$

2. $\frac{x^3}{3}+xy+e^y=c$

3. $\frac{1}{y}=-x\int^{x}\frac{e^{2t}}{t^2}dt+cx$, $y=0$

4. $2\arcsin\left(\frac{y}{x}\right)-\ln|x|=c$

5. $x+2\ln|y|-2\ln(\ln|x|)=c$

6. $u=x+y$, $x+y=\tan(x+c)$

7. $y^{-2}=ce^{2x^2}-4x^2-2$

8. $y=ux$, $y=x\sin\left[c\pm(x^2+y^2)/2\right]$, $x\lesseqgtr 0$

9. $\sin x=ce^{ay}-\left(\frac{y^2}{a}+\frac{2y}{a^2}+\frac{2}{a^3}\right)$

10. Ομογενής, $(1-cx)e^{y/x}=cx$

11. $u=xy$, χωρ. μετ., $xy=e^{cx}$

12. Bernoulli, ολ. παρ. $x^{-2}y^{-2}$, $x^2-y=cxy$

13. πλήρης, $3xy^2+x^2y+3y+x^2=c$

14. πλήρης, $x^2y^2-2(x+y)=c$

15. πλήρης, $x^3+y^3+3(x^2y+y+3x)=c$

16. Bernoulli, $2x^2=2y-1+ce^{-2y}$

17. $y=ux^2$, $(x^2+y)^2=cy$

18. Bernoulli, $y^2=cx-3x^2$

19. Ολ. παρ. $x^{-2}y^{-2}$, $x^3+y^3=cxy$

20. $u=xy$, $y^2=c\exp(xy-x^{-1}y^{-1})$

21. Ομογενής, πλήρης, $y^3+4x^3+3x^2y=c$

22. Bernoulli, $xy^3 = x^2 + c$ **23.** Ολοκλ. παραγ. $x^2 + y^2$, $(x^2 + y^2)^2 (2y^2 - x^2) = c$

24. Ολοκλ. παραγ. $x^{-1}y^{-2}$, $y^3 + y\ln|xy| - x = cy$

25. χωρ. μετ., πλήρης, $y^4 + y^2 = x^4 + x^2 + c$ **26.** $u = e^y$, $e^y = 1 + ce^{-e^x}$

27. $z = x + y - 2$, $y = 2 - x + \tan(x - 1 + \pi/4)$

28. Πλήρης $xy - 3x^2/2 - 8x - 2y = c$ **29.** χωρ., μετ., $y = \sin\left(x + \frac{\pi}{4}\right)$

30. γραμμική ως προς y', $y = (x-1)^3/6 + c\ln|x-1| + k$

31. $y_1 = -2$, $y = -2 + (ce^{-x} - 1)^{-1}$ **32.** $y_1 = \frac{e^x}{y}$, $y = e^x + 2(ce^{-3x} - e^x)^{-1}$

39. $M = x^2y + c$ **40.** $M = 2x^{-1}y^{-3} - \frac{3}{2}x^2y^{-4} + c$

41. ~~A~~ $= \frac{3}{2}$; $2x^3 + 9x^2y + 12y^2 = c$ **42.** $A = -2$, $2x^2 - 2y^2 - x = c\,xy^2$

45. $x(y^2 + 1) = cy$ **46.** $x^3y(2x - 3y) = c$

47. $(2x^3 - y)^2 = cyx^6$ **48.** $y^2(c - x) = x^3$

49. $(x + 2y - 1)^2 = 2y + c$ **50.** $5x^2y^3 = x^5 + 4$

51. $2y^2 = x^2(3x - 1)$ **52.** $(x - 3)^2(x - 3y) = c$

53. $x^2 + x - 3xy - y^2 + 4y = c$ **54.** $b^2y = c_1e^{bx} - abx - a - cb$

55. $\ln|x - y + 1| = c - x$