

MR3095325 (Review) 55P10 13A50

Kechagias, Nondas E. (GR-IOAN)

Dickson invariants and a new description of $H^*(Q_0S^0, \mathbb{Z}/p\mathbb{Z})$ via $H^*(B\Sigma_\infty, \mathbb{Z}/p\mathbb{Z})$.
 (English summary)

J. Homotopy Relat. Struct. **8** (2013), no. 2, 201–229.

Let $Q = \Omega^\infty \Sigma^\infty$ and Q_0S^0 be the base point component of QS^0 , where S^0 is the zero sphere. All cohomology will have $\mathbb{Z}/p\mathbb{Z}$ coefficients for a prime p . It was shown by D. G. Quillen [Ann. of Math. (2) **90** (1969), 205–295; [MR0258031 \(41 #2678\)](#)], and by M. Barratt and S. B. Priddy [Comment. Math. Helv. **47** (1972), 1–14; [MR0314940 \(47 #3489\)](#)], that Q_0S^0 and $B\Sigma_\infty$, the classifying space of the infinite symmetric group, have isomorphic cohomology. In his main result, the author gives explicit polynomial generators for $H^*(B\Sigma_\infty)$ in terms of Dickson invariants. The Dickson algebra is $P[y_1, \dots, y_k]^{\text{GL}_k} = P[d_{k,1}, \dots, d_{k,k}]$, the extended Dickson algebra is $D_k = (E_k(x_1, \dots, x_k) \otimes P[y_1, \dots, y_k])^{\text{GL}_k}$ and $D = \bigoplus_{k \geq 1} D_k^+$ is a module over the Steenrod algebra, \mathcal{A} .

For $p = 2$, the paper's main result becomes: $H^*(B\Sigma_\infty)$ is isomorphic to the polynomial algebra generated by $\{\prod_{1 \leq r \leq k} d_{k,k-r}^{m_r} \mid (m_1, \dots, m_k) \notin (2\mathbb{N})^k \text{ for } k \geq 1\}$.

In Theorem 6, an isomorphism between a subalgebra, SD_k , of D_k and the dual of the Dyer-Lashof operations of length k is recalled and Theorem 9 gives an algebra isomorphism between $H^*(Q_0S^0)$ and the universal enveloping algebra, $V(SD^\#)$, of the abelian restricted Lie algebra constructed from a submodule, SD , of D . The main result is proved using an isomorphism between a quotient of SD and the dual of a submodule of the Dyer-Lashof algebra. It is noted that the isomorphism of the main result is not one of \mathcal{A} -modules.

In the final section, a filtration of $V(SD^\#)$ consisting of \mathcal{A} -modules is used to re-prove a result of H. E. A. E. Campbell, F. P. Peterson and P. S. Selick [Trans. Amer. Math. Soc. **293** (1986), no. 1, 1–39; [MR0814910 \(87e:55010a\)](#)] that Q_0S^0 is H -atomic:

Let $f: Q_0S^0 \rightarrow Q_0S^0$ be an H -map such that $f^*: H^*(Q_0S^0) \rightarrow H^*(Q_0S^0)$ is a Steenrod coalgebra map which induces an isomorphism on degrees $2p - 3$ and $2(p^2 - 1)$. Then f^* is an isomorphism.

Zafer Mahmud

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[MR2520991](#) (2010h:20117) [20J05](#) [13A50](#) [18G10](#) [55S10](#)

Kechagias, Nondas E. (GR-IOAN)

The transfer in mod- P group cohomology between $\Sigma_P \int \Sigma_{P^{N-1}}$, $\Sigma_{P^{N-1}} \int \Sigma_P$ and Σ_{P^N} . (English summary)

J. Homotopy Relat. Struct. **4** (2009), no. 1, 153–179.

Let H be a subgroup of the finite group G , let p be a prime, and let $H^*(G)$ denote the group cohomology $H^*(G, \mathbf{Z}/p\mathbf{Z})$. The Weyl group $W_G(H)$ is $N_G(H)/HC_G(H)$, where $N_G(H)$ and $C_G(H)$ denote the normalizer and centralizer of H in G respectively, and the conjugation action of G on H induces an action of $W_G(H)$ on $H^*(H)$. Then we have the restriction map $(\text{res}_H^G)^*: H^*(G) \rightarrow H^*(H)^{W_G(H)}$, the $W_G(H)$ -invariants of $H^*(H)$, and the transfer map $\text{tr}^*: H^*(H) \rightarrow H^*(G)$.

If G is a subgroup of the symmetric group Σ_n on n elements and H is a finite group,

then $G \wr H$ denotes the wreath product, that is, the semidirect product of the normal subgroup H^n with G acting by permuting the factors. If also $H \leq \Sigma_m$, then $G \wr H$ is a subgroup of Σ_{mn} .

Let n be a positive integer, let $V = \mathbb{F}_p^n$, the n -dimensional vector space over the field with p elements, and view Σ_{p^n} as the symmetric group on V . Now V acts on itself by addition on the left, and this defines an inclusion $V \hookrightarrow \Sigma_{p^n}$. Also define $\Sigma_{p^n, p}$ to be $\mathbb{Z}/p\mathbb{Z} \wr \cdots \wr \mathbb{Z}/p\mathbb{Z}$ (n -factors). Then with the appropriate identifications, $\Sigma_{p^n, p}$ is a Sylow p -subgroup of Σ_{p^n} . Furthermore V is a maximal elementary abelian p -subgroup of $\Sigma_{p^n, p}$ that is contained in both $\Sigma_p \wr \Sigma_{p^{n-1}}$ and $\Sigma_{p^{n-1}} \wr \Sigma_p$.

If $V \leq G \leq \Sigma_{p^n}$, the transfer map $\text{tr}^*: H^*(G) \rightarrow H^*(\Sigma_{p^n})$ induces a map $\bar{\tau}^*: (\text{res}_V^G)^* \rightarrow (\text{res}_V^{\Sigma_{p^n}})^*$. In this paper, $\bar{\tau}^*$ is investigated for $G = \Sigma_{p^n, p}$, $\Sigma_{p^{n-1}} \wr \Sigma_p$, or $\Sigma_p \wr \Sigma_{p^{n-1}}$. The results are fairly technical and several interesting techniques are used. In particular, let D_n denote the Dickson algebra $(\mathbb{F}_p[y_1, \dots, y_n])^{\text{GL}(n, \mathbb{F}_p)}$. Then computations of free D_n -module bases of $(\mathbb{F}_p[y_1, \dots, y_n])^P$ for certain parabolic subgroups of P of $\text{GL}(n, \mathbb{F}_p)$ are important in the proofs.

Peter A. Linnell

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[Kechagias, Nondas E.](#) (GR-IOAN)

A note on parabolic subgroups and the Steenrod algebra action. (English summary)

Algebra Colloq. **15** (2008), no. 4, 689–698.

Let p be a prime, \mathbf{F}_p the field with p elements, V the n -dimensional vector space over \mathbf{F}_p and $\mathbf{F}_p[V]$ the symmetric algebra on the vector space V^* dual to V . The general linear group GL_n and the mod- p Steenrod algebra \mathcal{A} act on $\mathbf{F}_p[V]$. Let I be a partition of n and let $P(I)$ be the parabolic subgroup of GL_n associated to I . In [N. J. Kuhn and S. A. Mitchell, Proc. Amer. Math. Soc. **96** (1986), no. 1, 1–6; [MR0813797 \(87h:20084\)](#)] and [T. J. Hewett, Duke Math. J. **82** (1996), no. 1, 91–102; [MR1387223 \(97c:13002\)](#)] the invariants in $\mathbf{F}_p[V]$ of the parabolic subgroups $P(I)$ are computed, but the resulting generating set of $\mathbf{F}_p[V]^{P(I)}$ is not closed under the action of the Steenrod algebra. In the paper under review the author provides a new system of generators for $\mathbf{F}_p[V]^{P(I)}$ which turns out to be closed under the \mathcal{A} -action. He also derives relations between the new generators and the old ones. The results obtained are then applied to make explicit, in some cases, the \mathcal{A} -action on the new generators of $\mathbf{F}_p[V]^{P(I)}$. *Adriana Ciampella*

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[MR2404615 \(2009b:53051\)](#) [53C21](#) [53C40](#) [57R19](#)

[Kechagias, Nondas E.](#) (GR-IOAN)

Even dimensional submanifolds of spheres with nonnegative curvature operator. (English summary)

Soochow J. Math. **33** (2007), no. 4, 939–948.

There are many curvature-related sphere theorems in the literature, asserting that a certain compact manifold is homeomorphic to a sphere, provided that some curvature invariant (e.g. the Ricci curvature) is bounded from below by a constant depending only on the dimension. Recently, T. Vlachos proved that if M^n is an odd n -dimensional compact oriented submanifold of the unit sphere S^{n+l} with mean curvature $|H|$ for

which

$$(*) \quad \text{Ric} > \frac{n(n-3)}{n-1} + \frac{n^2(n-3)}{(n-1)^2} |H|^2 +$$

$$\frac{n(n-3)}{(n-1)^2} |H| \sqrt{n^2 |H|^2 + n^2 - 1},$$

then M is homeomorphic to a sphere when $n > 3$ and diffeomorphic to a space form of positive sectional curvature when $n = 3$ [T. Vlachos, Proc. Amer. Math. Soc. **130** (2002), no. 1, 167–173 (electronic); [MR1855635 \(2003c:53083\)](#)].

In the paper under review, the author extends the above result to simply-connected compact even-dimensional submanifolds. More precisely, the following is proved: Let M be a simply-connected $n = 2m$ -dimensional compact oriented submanifold of the unit sphere S^{2m+1} with non-negative curvature operator whose Ricci curvature satisfies $(*)$. If $m > 2$, then M is homeomorphic to S^{2m} or $S^m \times S^m$. If $m = 2$, then M is homeomorphic to $\mathbb{C}P^2$, S^4 , or $S^2 \times S^2$. Here the non-negative curvature operator ρ means that its eigenvalues are nonnegative at each point $p \in M$ when ρ is viewed as a map $\rho: \wedge^2(T_p M) \rightarrow \wedge^2(T_p M)$. The proof is mostly topological, involving a sequence of well-known auxiliary topological results.

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MR2158511 (2006d:13005) 13A50

Hughes, I. P. (3-QEN); Kechagias, N. (GR-IOAN)

Rings of invariants of modular p -groups which are hypersurfaces. (English summary)

J. Algebra **291** (2005), no. 1, 72–89.

For a vector space V over a field F let $S = S(V)$ be the symmetric algebra of V ; all groups are supposed to be subgroups of $\text{GL}(V)$. Given a finite non-modular group L with polynomial invariant ring S^L and its subgroup H , containing the derived group of L , H. Nakajima [J. Algebra **80** (1983), no. 2, 279–294; [MR0691804 \(85e:20036\)](#)] established a criterion on H for S^H to be a hypersurface. The proof is essentially based on the result that if S^H is a hypersurface, then between H and L there is a group G whose invariant ring is polynomial and $S^H = S^G[b]$ for b in S^H .

The main theorem of the article under review describes necessary and sufficient conditions on H to ensure that $S^H = S^G[b]$, where G is a finite linear p -group over $F = \mathbb{F}_p$ with polynomial invariant ring S^G and a subgroup H of G contains the derived group $[G, G]$.

Artem A. Lopatin

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MR2066463 (2005b:55014) [55P35](#) [13A50](#) [55S10](#)

Kechagias, N. E. (GR-IOAN)

An invariant theoretic description of the primitive elements of the mod- p cohomology of a finite loop space which are annihilated by Steenrod operations.
(English summary)

Invariant theory in all characteristics, 159–174, *CRM Proc. Lecture Notes*, 35, Amer. Math. Soc., Providence, RI, 2004.

The author gives an invariant-theoretic description, via the Dickson algebra, of the primitives $PH^*(\Omega_0^{2n+1}S^{2n+1}; \mathbf{Z}/p\mathbf{Z})$, and explicit formulas for which elements are annihilated by the Steenrod algebra. This presents an alternative approach to work of H. E. A. E. Campbell, F. P. Peterson and P. S. Selick [Trans. Amer. Math. Soc. **293** (1986), no. 1, 1–39; [MR0814910 \(87e:55010a\)](#)].

{For the entire collection see [MR2066574 \(2004m:13002\)](#)}

Donald M. Davis

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Kechagias, Nondas E. (GR-IOAN)

Adem relations in the Dyer-Lashof algebra and modular invariants. (English summary)

Algebr. Geom. Topol. **4** (2004), 219–241 (*electronic*).

The relationship between the Dyer-Lashof algebra $R[k]$ and the Dickson invariants $D[k]$ is well known.

In this paper, the author studies the Adem relations in the Dyer-Lashof algebra from a modular invariant point of view. In particular, he obtains an algorithm for calculating the hom-dual of an element in the Dyer-Lashof algebra and for finding the image of a non-admissible element after applying Adem relations. The key ingredient for relating homology operations and polynomial invariants is the relation between the map which imposes Adem relations and the decomposition map between certain rings of invariants. This relation was studied by Mui for $p = 2$ and the author extends it for any prime p . As an application, one can deal with polynomials instead of homology operations and one also has a moderate explanation of the complexity of Adem relations.

Kohhei Yamaguchi

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MR1738679 (2001c:55011) 55S12 55P47

Campbell, H. E. A. (3-QEN); Kechagias, N. E. (GR-IOAN)

Arnon completion of the Dyer-Lashof algebra. (English summary)

Israel J. Math. **114** (1999), 189–203.

Let \mathcal{A} be the mod 2 Steenrod algebra and $\mathcal{A}_\mu = \varprojlim(\frac{1}{2^t}\mathcal{A}, \mu)$ the complete Steenrod algebra defined by P. Arnon [“Generalized Dickson invariants”, Ph.D. Thesis, Massachusetts Inst. Tech., Cambridge, MA, 1994]. Here $\mu: \frac{1}{2^{t+1}}\mathcal{A} \rightarrow \frac{1}{2^t}\mathcal{A}$ denotes the halving homomorphism given by $\mu(\text{Sq}^{2^n}) = \text{Sq}^n$, $\mu(\text{Sq}^{2^{n+1}}) = 0$. Let D_k be the Dickson algebra of $\text{GL}(k, F_2)$ -invariants in $F_2[x_1, \dots, x_k]$ with $|x_i| = 1$. Arnon also defined the generalized Dickson algebra $D_k^\vee = \varprojlim(\frac{1}{2^t}D_k, \delta)$, where $\delta: \frac{1}{2^t}D_k \rightarrow \frac{1}{2^{t+1}}D_k$ is the Frobenius homomorphism given by $\delta(x) = x^2$. Roughly speaking, D_k^\vee is defined by attaching to D_k all 2-powers-th roots of elements in D_k .

In a previous paper [in *Algebraic topology: new trends in localization and periodicity (Sant Feliu de Guíxols, 1994)*, 271–284, Progr. Math., 136, Birkhäuser, Basel, 1996; **MR1397738 (97i:55032)**], I. Llerena and the reviewer defined a monomorphism $i_k: D_k^\vee \rightarrow D_{k+1}^\vee$ and noted that D_k cannot be embedded into D_{k+1} . This is the key point for them, to show that \mathcal{A}_μ is dual to $D_{\text{fin}}^\vee = \varinjlim D_k^\vee$. (In their notation, D_{fin}^\vee is denoted by D_∞^\vee .)

The paper under review develops Arnon’s and Llerena-Hung’s ideas for the Dyer-Lashof algebra R . The starting point of the paper is a well-known result by I. Madsen claiming that D_k is dual to the subcoalgebra $R[k]$ consisting of Dyer-Lashof operations of length k . The authors define the completion $R_\mu[k] = \varprojlim(\frac{1}{2^t}R[k], \mu)$, where $\mu: \frac{1}{2^{t+1}}R[k] \rightarrow \frac{1}{2^t}R[k]$ is the halving homomorphism given by $\mu(Q^{2^n}) = Q^n$, $\mu(Q^{2^{n+1}}) = 0$. Then, they set $\widehat{R}_\mu = \varinjlim(R_\mu[k], \theta)$, where $\theta: R_\mu[k] \rightarrow R_\mu[k+1]$ is defined by $\theta(Q^I) = Q^{(2^I, 0)}$. The main result of the paper states that \widehat{R}_μ is dual to $D^\vee = \varprojlim(D_k^\vee, \kappa)$, where $\kappa: D_{k+1}^\vee \rightarrow D_k^\vee$ is a canonical epimorphism which splits the above monomorphism $i_k: D_k^\vee \rightarrow D_{k+1}^\vee$.

Combined with Llerena and Hung’s result, it shows that $(\widehat{R}_\mu)^*$ contains a dense subalgebra isomorphic to \mathcal{A}_μ^* .

{Reviewer’s remark: The reviewer would like to remind the authors that the second paper cited above is not by the reviewer alone but a joint work by Llerena and himself.}

Nguyen H. V. Hung

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[MR1622345 \(99f:13003\)](#) [13A50](#) [18G10](#) [20G40](#) [55S10](#)

[Kechagias, Nondas E.](#) (GR-IOAN)

The transfer between rings of modular invariants of subgroups of $GL(n, p)$.
 (English summary)

Stable and unstable homotopy (Toronto, ON, 1996), 165–180, *Fields Inst. Commun.*, 19, Amer. Math. Soc., Providence, RI, 1998.

Let \mathbf{F}_p be the field having p elements and let V be an n -dimensional vector space over \mathbf{F}_p . Let S_n be the symmetric graded algebra on V , say $S_n = \mathbf{F}_p[y_1, \dots, y_n]$, and let $E_n = \mathbf{F}_p[x_1, \dots, x_n]$ be the graded exterior algebra on V when $p > 2$. The general linear group, $GL_n(\mathbf{F}_p)$ acts on S_n and E_n . The author considers various algebras of invariants associated to these actions. In particular, let U_n be the subgroup of $GL_n(\mathbf{F}_p)$ consisting of all upper triangular matrices having 1's on the main diagonal. Let D_n [resp. H_n] be the algebra of $GL_n(\mathbf{F}_p)$ -invariants [resp. U_n -invariants] of S_n . The algebra S_n is a finitely generated, free H_n -module and the author gives explicit formulas for basis elements in terms of the y_i 's. (Campbell and Hughes have carried out a similar program.) Bases are also described for S_n and H_n as D_n -modules. In the latter situation, an algorithm is given for writing an element in H_n as a linear combination of the basis elements. Analogous facts are proved for the actions of $GL_n(\mathbf{F}_p)$ and U_n on $E_n \otimes S_n$. Again, the algebra of U_n -invariants is free over D_n and the author describes a basis explicitly in terms of the y_i 's, x_j 's and the generators of H_n . In this context, he shows that the transfer map and natural projection (relative to the given basis) from the algebra of U_n -invariants to the algebra of $GL_n(\mathbf{F}_p)$ -invariants agree on the ideal generated by $d_{n,0}$. Throughout the paper, the author discusses the relationship between these topics and the mod- p cohomology of subgroups of the symmetric group.

{For the entire collection see [MR1622333 \(99a:55001\)](#)}

Frank D. Grosshans

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MR1443725 (98c:13009) 13A50 20H30

Kechagias, Nondas E. (GR-AEG2)

A note on $U_n \times U_m$ modular invariants. (English summary)

Canad. Math. Bull. **40** (1997), no. 1, 54–59.

For a fixed prime p , let U_n denote the group of upper triangular $n \times n$ matrices with all diagonal entries equal to one, over the field \mathbf{F}_p of cardinality p .

This paper presents a description of the invariants of the group $U_n \times U_m$ acting (in the usual way) on the space of $n \times m$ matrices over the field \mathbf{F}_p . *Joseph P. Brennan*

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MR1617511 (99b:55035) 55S05 13A50 20J05 55R12

Kechagias, Nondas E. (GR-IOAN)

Modular invariant theory, homology operations, and the transfer between rings of invariants of parabolic subgroups. (English summary)

Proceedings of the 1st Panhellenic Algebra Conference (Athens, 1996).

Bull. Greek Math. Soc. **38** (1996), 79–88.

Summary: “The relation between modular invariant theory and algebraic topology is described, namely, the role of Dickson invariants in homology operations and the associated transfer between certain rings of invariants. The transfer map in cohomology is explicitly discussed for parabolic subgroups.”

{For the entire collection see [MR1617504 \(99a:00037\)](#)}

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MR1291121 (95g:55007) 55S12 20G10

[Kechagias, Nondas E.](#) (3-YORK-MS)

Extended Dyer-Lashof algebras and modular coinvariants. (English summary)

Manuscripta Math. 84 (1994), no. 3-4, 261–290.

Given a pointed space X , it is well known that the spaces $E\Sigma_n \times_{\Sigma_n} X^n$, for $n = 1, 2, \dots$, can be assembled into a space CX , and that the homology of CX is a well-understood functor of the homology of X [see e.g., F. R. Cohen, T. J. Lada, and J. P. May, *The homology of iterated loop spaces*, Lecture Notes in Math., 533, Springer, Berlin, 1976; [MR0436146 \(55 #9096\)](#)]. The space CX is of interest as it is naturally homotopy equivalent to $\Omega^\infty \Sigma^\infty X$ if X is connected.

In the paper at hand, the author notes that an increasing sequence of integers, $N = (n_1, n_2, \dots)$, can be used to define certain subgroups $G_n(N)$ of Σ_n , each containing the p -Sylow subgroup of Σ_n , and that the spaces $E\Sigma_n \times_{G_n(N)} X^n$ can be assembled into a space $G_N X$. The homology of $G_N X$ is a functor of the homology of X . *N. J. Kuhn*

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MR1227660 (94i:55026) 55S12

[Kechagias, Nondas E.](#) (GR-CRET)

Mod odd modular coinvariants, homology operations, and limit spaces. (English summary)

Canad. J. Math. 45 (1993), no. 4, 803–819.

Summary: “We compute the homology of $\lim_{n \rightarrow \infty} (G_n \wr X)$, where (G_n) is a system of subgroups of Σ_{p^n} containing a p -Sylow subgroup $(\Sigma_{p^n, p})$ and satisfying certain properties. We show that $H_*(\lim_{n \rightarrow \infty} (G_n \wr X); \mathbf{Z}/p\mathbf{Z})$ is built naturally over homology operations related to (G_n) . We describe this family of operations using modular coinvariants.” *N. J. Kuhn*

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MR1152986 (93i:55023) 55S10

Kechagias, Nondas E. (3-QEN)

The Steenrod algebra action on generators of rings of invariants of subgroups of $GL_n(\mathbf{Z}/p\mathbf{Z})$.

Proc. Amer. Math. Soc. **118** (1993), no. 3, 943–952.

Let p be an odd prime and consider the mod p cohomology H^*A of an elementary abelian p -group of rank n . The group $GL(n, \mathbf{Z}/p)$ acts on H^*A by algebra automorphisms which commute with the action of the mod p Steenrod algebra.

The author starts by describing the computation of the invariant subalgebras $(H^*A)^G$ if G is a parabolic subgroup P of $GL(n, \mathbf{Z}/p)$. These results are taken from the author's Ph.D. thesis ["Homology operations and modular invariant theory", Ph.D. Thesis, Kingston, ON, 1990]. In case $G = GL(n, \mathbf{Z}/p)$ or $G = U_n$, the unipotent subgroup of $GL(n, \mathbf{Z}/p)$, the computations were done by Huỳnh Múi [J. Fac. Sci. Univ. Tokyo Sect. IA Math. **22** (1975), no. 3, 319–369; [MR0422451 \(54 #10440\)](#)]. Then the author treats the action of the reduced power operations and of the Bockstein homomorphism on the exhibited algebra generators of $(H^*A)^P$ and gives closed formulae for these actions. Numerous references to earlier related work by other authors are given.

Hans-Werner Henn

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