

COMPUTATIONAL FINANCE

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Exercise 1

Set-up. Let the dynamics of a stock price process $S = (S_t)_{t \geq 0}$ evolve according to the Black–Scholes model, i.e.

$$S_t = S_0 \exp\left(\sigma W_t + \left(r - \frac{1}{2}\sigma^2\right)t\right), \quad S_0 > 0, \quad (1)$$

where $W = (W_t)_{t \geq 0}$ denotes a standard Brownian motion. Consider the *arithmetic* Asian option with payoff function

$$\left(\frac{1}{n} \sum_{i=1}^n S_{t_i} - K\right)^+,$$

and the (artificial) *geometric* Asian option with payoff function

$$\left(\left[\prod_{i=1}^n S_{t_i}\right]^{1/n} - K\right)^+,$$

where $T_n = T$ denotes the maturity of the options.

Tasks.

- (1) Derive an explicit formula for the price of the geometric Asian option.
- (2) Simulate the price of the arithmetic Asian option using:
 - (plain) Monte Carlo method
 - Monte Carlo with antithetic variates
 - Monte Carlo with control variates, using the geometric Asian option as control.
- (3) Compare the results in terms of accuracy and efficiency. In particular, plot the root mean square error and the computational time as functions of the number of simulations M and compare with the theoretical convergence rate.
- (4) Finally, combine both variance reduction techniques. Is there a further effect?

Data.

- Spot price $S_0 = 100$, strike price $K = 100$, volatility $\sigma = 25\%$.
- Maturity $T = 1$, $n = 365$, interest rate $r = 2\%$.

Submit.

- The source code (in `scilab/matlab/C/python/...`). The source code should include sufficient documentation.
- A PDF file explaining how the code was developed and discussing the results (preferably written in \LaTeX).
- Submit everything per e-mail to
christian.bayer@wias-berlin.de and papapan@math.tu-berlin.de
in a zip file named: `Exercise_1.Surname_Name`.
- Deadline: **May 29, 2016**.