COMPUTATIONAL FINANCE

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Exercise

Set-up. Let the dynamics of a stock price process $S = (S_t)_{t\geq 0}$ evolve according to the Black–Scholes model, i.e.

$$S_t = S_0 \exp\left(\sigma W_t + \big(r - \frac{1}{2}\sigma^2\big)t\right), \quad S_0 > 0, \tag{1}$$

where $W = (W_t)_{t\geq 0}$ denotes a standard Brownian motion. Consider the *arithmetic* Asian option with payoff function

$$\left(\frac{1}{n}\sum_{i=1}^{n} S_{t_i} - K\right)^+,$$

and the (artificial) *geometric* Asian option with payoff function

$$\left(\left[\prod_{i=1}^{n} S_{t_i}\right]^{1/n} - K\right)^+,$$

where $T_n = T$ denotes the maturity of the options.

Tasks.

(1) Derive an explicit formula for the price of the geometric Asian option.
(2) Simulate the price of the arithmetic Asian option using:
  - (plain) Monte Carlo method
  - Monte Carlo with antithetic variates
  - Monte Carlo with control variates, using the geometric Asian option as control.

(3) Compare the results in terms of accuracy and efficiency. In particular, plot the root mean square error and the computational time as functions of the number of simulations M and compare with the theoretical convergence rate.
(4) Finally, combine both variance reduction techniques. Is there a further effect?

Data.

- Spot price $S_0 = 100$, strike price $K = 100$, volatility $\sigma = 25\%$.
- Maturity $T = 1$, $n = 365$, interest rate $r = 2\%$.

Submit.

- The source code (in `scilab`/`matlab`/`C`/...). The source code should include sufficient documentation.
- A PDF file explaining how the code was developed and discussing the results (preferably written in LaTeX).
- Submit everything per e-mail to papapan@math.tu-berlin.de in a zip file named: `Exercise_Surname_Name`.
- Deadline: **May 31, 2015**.