

# COMPUTATIONAL FINANCE

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## Exercise 2

**Set-up.** Consider the Heston stochastic volatility model (under a risk neutral measure), provided by the SDEs

$$\begin{aligned}dX_t &= \left(r - \frac{1}{2}V_t\right)dt + \sqrt{V_t}dW_t, & X_0 &= x, \\dV_t &= \kappa(\theta - V_t)dt + \eta\sqrt{V_t}d\bar{W}_t, & V_0 &= v.\end{aligned}\tag{1}$$

The parameters satisfy  $r \in \mathbb{R}$ ,  $\kappa, \theta, \eta \in \mathbb{R}_+$ , the initial values are  $x \in \mathbb{R}, v \in \mathbb{R}_+$ , and the Brownian motions  $W, \bar{W}$  are correlated with parameter  $\rho \in [-1, 1]$ . An Euler discretization of this SDE was provided in the class.

### Tasks.

- (1) Compute the prices of a call option and the 95% confidence intervals with the Euler Monte Carlo method.
- (2) Study empirically the convergence of the Euler scheme.

### Data.

- Spot price  $S_0 = 100$ , interest rate  $r = 0\%$ .
- Maturity  $T = 5$ , strike prices  $K = \{80, 100, 120\}$ .
- Heston parameters:  $\kappa = 1$ ,  $\theta = v = 9\%$ ,  $\eta = 1$ ,  $\rho = -0.3$ .

### Submit.

- The source code (in `scilab/matlab/C/...`). The source code should include sufficient documentation.
- A PDF file explaining how the code was developed and discussing the results (preferably written in  $\text{\LaTeX}$ ).
- Submit everything per e-mail to `Christian.Bayer@wias-berlin.de` and `papapan@math.tu-berlin.de` in a zip file named: `Exercise_2.Surname_Name`.
- Deadline: **June 28, 2013**

### Hints.

- For the Heston model, there is no closed-form expression for the option prices. Thus, you will first have to compute a sufficiently precise reference price using the Euler Monte Carlo method with  $N_{\text{ref}}$  (number of steps) and  $M_{\text{ref}}$  (number of trajectories) (very) large.<sup>1</sup>
- Choose a sequence of  $N_i$  and corresponding  $M_i$  and consider the corresponding weak errors, i.e., the (absolute) differences of the option prices obtained with  $(N_i, M_i)$  and the reference price obtained previously. Compute the “empirical rate of convergence”, i.e., determine the parameters  $C$  and (above all)  $\alpha$  in a regression  $\text{error}_i \sim CN_i^{-\alpha}$ .

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<sup>1</sup>You will have to determine what this means. It may well be that your subsequent error analysis will show that the original choice of  $N_{\text{ref}}$  or  $M_{\text{ref}}$  was not big enough. Then you will have to restart the experiments with new choices for  $N_{\text{ref}}$  and/or  $M_{\text{ref}}$ .

- As we want to study the convergence of the discretization error, we need to choose  $M_i$  such that the Monte Carlo error is significantly smaller than the discretization error. In other words, the confidence interval around  $\text{error}_i$  should be considerably smaller than  $\text{error}_i$  itself.<sup>2</sup>
- Plot your observed errors  $\text{error}_i$  together with the confidence intervals around them and with the reference curve  $CN_i^{-\alpha}$ . As usual, you should use a log-log-plot. You might use Figure 3.5 in the lecture notes as an example of how your plot could look like. The plot gives you a visual check of whether the  $M_i$  have been sufficiently large. (Figure 3.5 is an extreme case, though. You should not try to make the confidence interval comparably small in the homework!)
- The submitted pdf-file should contain the plot as well as the underlying data and the estimated parameters  $C$  and  $\alpha$ .
- Do not throw away your code and, in particular, your reference price. You will need them for the third homework, as well.

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<sup>2</sup>This means that you can practically rule out that the Monte Carlo error has a significant effect on the estimated  $\alpha$ .