COMPUTATIONAL FINANCE

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Exercise 1

Set-up. Let the dynamics of a stock price process $S = (S_t)_{t\geq 0}$ evolve according to the Black–Scholes model, i.e.

$$S_t = S_0 \exp\left(\sigma W_t + \big(r - \frac{1}{2}\sigma^2\big)t\right), \quad S_0 > 0, \tag{1}$$

where $W = (W_t)_{t\geq 0}$ denotes a standard Brownian motion. Consider the *arithmetic* Asian option with payoff function

$$\left(\frac{1}{n}\sum_{i=1}^{n} S_{t_i} - K\right)^+,$$

and the (artificial) *geometric* Asian option with payoff function

$$\left(\left[\prod_{i=1}^{n} S_{t_i}\right]^{1/n} - K\right)^+,$$

where $T_n = T$ denotes the maturity of the options.

Tasks.

(1) Derive an explicit formula for the price of the geometric Asian option.
(2) Simulate the price of the arithmetic Asian option using:
  - (plain) Monte Carlo method
  - Monte Carlo with antithetic variates
  - Monte Carlo with control variates, using the geometric Asian option as control.

(3) Compare the results in terms of accuracy and efficiency (computational time).
(4) Finally, combine both variance reduction techniques. Is there a further effect?

Data.

- Spot price $S_0 = 100$, strike price $K = 100$, volatility $\sigma = 25\%$, interest rate $r = 2\%$.
- Maturity $T = 1$, $n = 365$.

Submit.

- The source code (in `scilab`/`matlab`/`C`, ...).
- A PDF file explaining how the code was developed and discussing the results (preferably written in LaTeX).