Dual algebraic structures and enrichment

Christina Vasilakopoulou

National Technical University of Athens, Greece

Categories and Companions Symposium

20 September 2022

Outline

1. Background
2. Sweedler theory for (co)monoids and (co)modules
3. Many-object generalization
4. Further directions

Algebras and coalgebras

Suppose $(\mathcal{V}, \otimes, I)$ is monoidal category.

A category with a *tensor* product $\otimes \colon \mathcal{V} \times \mathcal{V} \to \mathcal{V}$ and a *unit* $I \in \mathcal{V}$ such that $(X \otimes Y) \otimes Z \cong X \otimes (Y \otimes Z)$ and $I \otimes X \cong X \cong X \otimes I$ coherently.

- A *monoid* is an object A together with maps $\mu \colon A \otimes A \to A$ and $\eta \colon I \to A$ which are associative and unital:

$$\begin{tikzcd} A \otimes A \otimes A \arrow[r, "\mu\otimes 1"] \arrow[d, "1\otimes\mu"'] & A \otimes A \arrow[d, "\mu"] \\ A \otimes A \arrow[r, "\mu"] & A \end{tikzcd} \qquad \begin{tikzcd} I \otimes A \arrow[r, "\eta\otimes 1"] \arrow[dr] & A \otimes A \arrow[d, "\mu"] & A \otimes I \arrow[l, "1\otimes\eta"'] \arrow[dl] \\ & A & \end{tikzcd}$$

$\star$ In $(\mathsf{Ab}, \otimes, \mathbb{Z})$, rings; in $(\mathsf{Vect}_\mathsf{k}, \otimes, k)$, k-algebras; in $(\mathsf{Cat}, \times, \mathbf{1})$, *strict* monoidal categories!

Its dual notion? A monoid in $\mathcal{V}^{\mathrm{op}}$...

- A *comonoid* is an object C together with maps $\delta\colon C \to C \otimes C$ and $\epsilon\colon C \to I$ which are coassociative and counital:

$$
\begin{tikzcd}
C \otimes C \otimes C & C \otimes C \arrow[l, "\delta\otimes 1"'] \\
C \otimes C \arrow[u, "1\otimes\delta"] \arrow[ur, phantom, "\delta(c)=\sum_{(c)} c_1\otimes c_2"] & C \arrow[l, "\delta"] \arrow[u, "\delta"']
\end{tikzcd}
\qquad
\begin{tikzcd}
I \otimes C & C \otimes C \arrow[l, "\epsilon\otimes 1"'] \arrow[r, "1\otimes\epsilon"] & C \otimes I \\
& C \arrow[ul] \arrow[u, "\delta"] \arrow[ur] &
\end{tikzcd}
$$

$\star$ In $(\mathrm{Cat}, \times, \mathbf{1})$, any category! With $\delta(X)=(X,X)$ and $\epsilon(X)=*$ "trivially".

$\star$ In $(\mathrm{Mod}_R, \otimes_R, R)$, R-coalgebras: divided power coalgebra, group-like coalgebra, trigonometric coalgebra...

Monoids and comonoids in $(\mathcal{V}, \otimes, I)$, together with maps that preserve (co)multiplication and (co)units, form categories Mon and Comon.

Suppose $(\mathcal{V}, \otimes, I)$ is symmetric, with $\sigma_{XY} \colon X \otimes Y \cong Y \otimes X$.

Mon and Comon are themselves monoidal , with I and

$$A \otimes B \otimes A \otimes B \xrightarrow{1 \otimes \sigma \otimes 1} A \otimes A \otimes B \otimes B \xrightarrow{\mu \otimes \mu} A \otimes B$$

Suppose $(\mathcal{V}, \otimes, I, \sigma)$ is monoidal closed, with $- \otimes X \dashv [X, -]$ for all X.

For any comonoid C and monoid A, $[C, A]$ is a monoid via *convolution*

$[C, A] \otimes [C, A] \to [C, A]$ which under tensor-hom adj. is

$$\begin{tikzcd}
{[C,A]\otimes[C,A]\otimes C} \arrow[r, "1\otimes\delta"] \arrow[drrr, dashed, bend right, "(f*g)(c)=\sum_{(c)} f(c_1)g(c_2)"'] & {[C,A]\otimes[C,A]\otimes C\otimes C} \arrow[r, "1\otimes\sigma\otimes1"] & {[C,A]\otimes C\otimes[C,A]\otimes C} \arrow[d, "\mathrm{ev}\otimes\mathrm{ev}"] \\
& & A \otimes A \arrow[d, "\mu"] \\
& & A
\end{tikzcd}$$

Sweedler theory: Motivation

Idea 0: For any k-coalgebra C, its linear dual $C^* = \mathrm{Hom}_k(C, k)$ is a k-algebra via convolution. For any k-algebra A, A^* is a coalgebra only when it is finite-dimensional. Find an operation that 'fixes' that?

Idea 1: [Sweedler, 1969] For any three vector spaces A, B and C,

$$\mathrm{Hom}(C \otimes B, A) \cong \mathrm{Hom}(B, \mathrm{Hom}(C, A)).$$

If C coalgebra, A, B algebras, when is it an *algebra* map $B \to \mathrm{Hom}(C, A)$?

Answer (low-level): when $f \colon C \otimes B \to A$ *measures*, i.e. satisfies

$$f(c \otimes aa') = \sum f(c_{(1)} \otimes a) f(c_{(2)} \otimes a')$$
$$f(c \otimes 1) = \epsilon(c) 1$$

There exists a *universal measuring* coalgebra P, namely for any other measuring coalgebra C, we get a unique coalgebra map $P \to C$.

$\star$ Constructed bijection from $\mathsf{Alg}(B, \mathrm{Hom}(C, A))$ to $\mathsf{Coalg}(C, P)$, where $P{=}P(A, B)$ is sum of subcoalgebras of cofree coalgebra on $\mathrm{Hom}(A, B)$...

Answer (high-level): $\mathrm{Hom}(\text{-}, A)\colon \mathsf{Coalg}^{\mathrm{op}} \to \mathsf{Alg}$ has an adjoint $P(A, \text{-})$!

Special case of more general result. . . for *locally presentable* categories.

A category is locally presentable when it has all colimits, and all objects are (λ-)filtered colimits of a set of certain presentable objects.

$\star$ From Vect_k, move to dgVect, gVect, Mod_R and many more!

Suppose $\mathcal{V}$ is a symmetric monoidal closed and locally presentable category. There is a 'parameterized' adjunction between

$$[\text{-}, \text{-}]\colon \mathsf{Comon}^{\mathrm{op}} \times \mathsf{Mon} \to \mathsf{Mon} \quad \text{given - convolution}$$
$$P(\text{-}, \text{-})\colon \mathsf{Mon}^{\mathrm{op}} \times \mathsf{Mon} \to \mathsf{Comon} \quad \text{new - universal measuring}$$

Sweedler theory for monoidal categories

- [Anel-Joyal, 2013]: dgVect_k, functors related to bar-cobar construction
  - convolution $[\text{-},\text{-}]$ and 'Sweedler hom' $P(\text{-},\text{-})$
  - 'Sweedler product' $N(\text{-},\text{-})\colon \mathrm{Comon}\times\mathrm{Mon}\to\mathrm{Mon}$ w. $N(C,\text{-})\dashv[C,\text{-}]$
- In fully general setting, universal measuring comonoid is

$$P(A,B)=\left(\mathrm{Lan}_{[\text{-},B]}\mathbf{1}_{\mathrm{Comon}}\right)(A)=\int^{C}\mathrm{Mon}(A,[C,B])\cdot C$$

$\star$ For $\mathcal{V}$=Set, Comon $\cong$ Set and the set $P(A,B)$ is $\mathrm{Mon}(A,B)$.

$\star$ Low-level is special case $\mathcal{V}=\mathrm{Vect}_k$, also Idea 0: *finite Sweedler dual*

$$A^o=\{f\in A^*\mid \ker f\,\text{contains cofinite ideal}\}$$

now expressed as $A^0=P(A,I)$ for which $\mathrm{Alg}(A,C^*)\cong\mathrm{Coalg}(C,A^o)$.

$\star$ 'Generalized algebra maps': $P(A,B)$ contains k-algebra maps, as the group-like elements $\delta(f)=f\otimes f$.

Enrichment of algebras in coalgebras

[Wraith, 1970's] k-algebras are enriched in k-coalgebras...

A $(\mathcal{V}, \otimes, I)$-enriched category has objects $\mathrm{ob}\mathcal{C}$ and hom-objects $\mathcal{C}(x, y) \in \mathcal{V}$ with composition and identity rules as maps in $\mathcal{V}$

$$\mathcal{C}(x, y) \otimes \mathcal{C}(y, z) \to \mathcal{C}(x, z), \quad I \to \mathcal{C}(x, x)$$

$\star$ Linear=Vect_k-enriched; abelian$\sim$ Ab-enriched; 2-cats=Cat-enriched.

$\star$ A $\mathcal{V}$-category with a single object 'is' a monoid in $\mathcal{V}$!

Back to our setting, induced convolution $[\text{-}, \text{-}]$ has extra structure: it is an *action* of the monoidal category $\mathsf{Comon}^{(\mathrm{op})}$ on the category Mon!

Any parameterized adjoint of an action $\bullet\colon \mathcal{V} \times \mathcal{C} \to \mathcal{C}$ gives rise to a $\mathcal{V}$-enriched structure on $\mathcal{C}$; all 'tensored' $\mathcal{V}$-categories arise this way.

Desired enrichment in very general setting, putting all pieces together.

Suppose $\mathcal{V}$ is symmetric monoidal closed and locally presentable. The category Mon is enriched in the symmetric monoidal Comon.

Digression: theory of Hopf categories

- A *bimonoid* in $\mathcal{V}$ is a monoid and a comonoid in a compatible way ; a *Hopf monoid* is a bimonoid with antipode.
- Their many-object generalizations? *Semi-Hopf* and *Hopf* $\mathcal{V}$-categories.

In particular, a semi-Hopf $\mathcal{V}$-category comes with

· $H(x,y) \otimes H(y,z) \xrightarrow{m_{xyz}} H(z,x),\ I \xrightarrow{j_x} H(x,x)$ 'global' multipl

· $H(a,b) \xrightarrow{d_{ab}} H(a,b) \otimes H(a,b),\ H(a,b) \xrightarrow{e_{ab}} I$ 'local' comultipl

The category of monoids is a semi-Hopf $\mathcal{V}$-category.

Universal measuring comodules

[Batchelor, 1990's] (Universal) measuring comodules: motivation and construction follows Sweedler's, applications to algebra and geometry

⋆ Sketch high-level approach, involving *(co)modules* in arbitrary $(\mathcal{V}, \otimes, I)$

- For a monoid A, an A-module M comes with $\mu\colon A \otimes M \to M$ (associative and unital)
- For a comonoid C, a C-comodule X comes with $\chi\colon X \to C \otimes X$ (coassociative and counital)
- With maps preserving (co)actions, categories ${}_A\mathrm{Mod}$ and ${}_C\mathrm{Comod}$

■ 'Global' categories Mod, Comod of (co)modules for any (co)monoid, maps for Mod are $g\colon {}_AM \to {}_NB$ in $\mathcal{V}$ with $f\colon A \to B$ in $\mathrm{Mon}(\mathcal{V})$ that

$$\begin{array}{ccccc}
A \otimes M & & \xrightarrow{\qquad\mu\qquad} & & M \\
{\scriptstyle 1\otimes g}\downarrow & & & & \downarrow{\scriptstyle g} \\
A \otimes N & \xrightarrow{f\otimes 1} & B \otimes N & \xrightarrow{\mu} & N
\end{array}$$

⋆ In a symmetric monoidal closed $\mathcal{V}$, for any C-comodule X, A-module M, $[X, M]$ is a $[C, A]$-module!

Suppose $\mathcal{V}$ is a symmetric monoidal closed and locally presentable category. There is a parameterized adjunction between

$$[\text{-},\text{-}]\colon \mathsf{Comod}^{\mathrm{op}} \times \mathsf{Mod} \to \mathsf{Mod} \quad \text{given - convolution}$$

$$Q(\text{-},\text{-})\colon \mathsf{Mod}^{\mathrm{op}} \times \mathsf{Mod} \to \mathsf{Comod} \quad \text{new - universal measuring}$$

■ If $\mathcal{V}$ is symmetric, Comod is (symmetric) monoidal: for ${}_CX$ and ${}_DY$

$$X \otimes Y \xrightarrow{\chi\otimes\chi} C \otimes X \otimes D \otimes Y \xrightarrow{1\otimes\sigma\otimes 1} C \otimes D \otimes X \otimes Y$$

Warning: no fixed-comodule category ${}_C\mathsf{Comod}$ is monoidal in general!

⋆ The functor $[\text{-},\text{-}]$ is again an action from Comod on Mod. . .

Suppose $\mathcal{V}$ is a symmetric monoidal closed and locally presentable category. The category Mod is enriched in the symmetric monoidal Comod.

Enriched fibration

So far: enrichment of Mon in Comon, also of Mod in Comod

$\star$ Independently of the enrichments, in any monoidal $\mathcal{V}$ these categories form a *fibration* Mod $\to$ Mon and an *opfibration* Comod $\to$ Comon

A fibration is a functor $F\colon \mathcal{C} \to \mathcal{D}$ with lifting universal property: for any map $f\colon A \to B$ in $\mathcal{D}$, and any object M above B, exists unique map $f^*M \to M$ above f used to factor any suitable $N \to M$ in $\mathcal{C}$.

■ The general situation is captured by an *enriched fibration* structure

$$\begin{tikzcd}
\mathsf{Mod} \arrow[r, dashed, "\text{enriched}"] \arrow[d, "\text{fibration}"'] & \mathsf{Comod} \arrow[d, "\text{opfibration}"] \\
\mathsf{Mon} \arrow[r, dashed, "\text{enriched}"] & \mathsf{Comon}
\end{tikzcd}$$

Generalizing from one to many objects

Initially: monoids become categories, comonoids become *cocategories*!

- If $(\mathcal{V}, \otimes, I)$ has coproducts preserved by $\otimes$, a $\mathcal{V}$-cocategory has objects $\mathrm{ob}\mathcal{C}$ and hom-objects $\mathcal{C}(x,y) \in \mathcal{V}$ with coherent

$$C(x,z) \xrightarrow{d_{xyz}} \sum_y C(x,y) \otimes C(y,z) \qquad C(x,x) \xrightarrow{\epsilon_x} I$$

Note: *opcategories*, i.e. $\mathcal{V}^{\mathrm{op}}$-categories, are not as convenient formally...

$\mathcal{V}$-categories are monads & $\mathcal{V}$-cocategories are comonads in bicategory of $\mathcal{V}$-matrices: objects are sets, maps $S\colon X \nrightarrow Y$ are families $\{S(x,y)\} \in \mathcal{V}$,

$$(S \circ T)(x,z) = \sum_y T(x,y) \otimes S(y,z) \quad \text{composition is matrix mult}$$

$\star$ Pointer: the formed categories $\mathcal{V}$-Cat and $\mathcal{V}$-Cocat are better captured inside a *double* category of matrices...

Direction 1: Under running assumptions, $\mathcal{V}$-Cocat has good properties (sym mon closed, loc pres) that allow "universal measuring cocategories"

Suppose $\mathcal{V}$ is a symmetric monoidal closed and locally presentable category. There is a parameterized adjunction between

$$H(\text{-},\text{-})\colon \mathcal{V}\text{-Cocat}^{\mathrm{op}} \times \mathcal{V}\text{-Cat} \to \mathcal{V}\text{-Cat} \quad \text{given - `convolution'}$$
$$S(\text{-},\text{-})\colon \mathcal{V}\text{-Cat}^{\mathrm{op}} \times \mathcal{V}\text{-Cat} \to \mathcal{V}\text{-Cocat} \quad \text{new - universal measuring}$$

$H(C, A)$ consists of functions and $H(C,A)(f,g) = \prod_{x,y} [C(x,y), A(fx, gy)]$.

The category $\mathcal{V}$-Cat is enriched in the symmetric monoidal $\mathcal{V}$-Cocat.

Similar things happen for (co)modules for (co)categories...but behind technical results lies a clearer picture.

Direction 2: "push" necessary structure even further up!

Generalizing from monoidal to double categories

Idea: clarify necessary structure on double category $\mathcal{V}\text{-}\mathbb{M}\mathrm{at}$ & abstract!

- A double category $\mathbb{D}$ consists of
  - object category $\mathbb{D}_0$ (0-cells & vertical 1-cells)
  - arrow category $\mathbb{D}_1$ (horizontal 1-cells & 2-morphisms) $\begin{array}{ccc} X & \xrightarrow{A} & Y \\ {\scriptstyle f}\downarrow & \Downarrow\alpha & \downarrow{\scriptstyle g} \\ Z & \xrightarrow[B]{} & W \end{array}$
  - $\mathbb{D}_0 \xrightarrow{\mathbf{1}} \mathbb{D}_1$, $\mathbb{D}_1 \underset{\mathfrak{t}}{\overset{\mathfrak{s}}{\rightrightarrows}} \mathbb{D}_0$, $\mathbb{D}_1 \times_{\mathbb{D}_0} \mathbb{D}_1 \xrightarrow{\circ} \mathbb{D}_1$ plus coherent isomorphisms.

0-cells, horizontal 1-cells, *globular* 2-morphisms make bicategory $\mathcal{H}(\mathbb{D})$.

$\star$ For $\mathbb{D}=\mathcal{V}\text{-}\mathbb{M}\mathrm{at}$, $\mathcal{V}\text{-}\mathbb{M}\mathrm{at}_0=\mathrm{Set}$ & $\mathcal{H}(\mathcal{V}\text{-}\mathbb{M}\mathrm{at})$ the usual bicat of matrices

- A monad in $\mathbb{D}$ is $A\colon X \nrightarrow X$ with associative, unital

$$\begin{array}{ccccc} X & \xrightarrow{A} & X & \xrightarrow{A} & X \\ \| & & \Downarrow\mu & & \| \\ X & & \xrightarrow[A]{} & & X \end{array} \qquad \begin{array}{ccc} X & \xrightarrow{1_X} & X \\ \| & \Downarrow\eta & \| \\ X & \xrightarrow[A]{} & X \end{array}$$

For any double category $\mathbb{D}$, there are categories of (co)monads $\mathrm{Mnd}(\mathbb{D})$, $\mathrm{Cmd}(\mathbb{D})$ as well as global categories of (co)modules $\mathrm{Mod}(\mathbb{D})$, $\mathrm{Comod}(\mathbb{D})$.

⋆ $\mathrm{Mnd}(\mathcal{V}\text{-}\mathbb{M}\mathrm{at}) = \mathcal{V}\text{-}\mathrm{Cat}$ and $\mathrm{Cmd}(\mathcal{V}\text{-}\mathbb{M}\mathrm{at}) = \mathcal{V}\text{-}\mathrm{Cocat}$ as foreseen

- *Fibrant* double cats: vertical 1-cells turn to horizontal in a coherent way

⋆ Function $f\colon X \to Y$ gives matrices $f^*(x,y) = f_!(y,x) = \begin{cases} I \text{ if } fx = y \\ 0 \text{ if } fx \neq y \end{cases}$

- *Monoidal* double cats: $\mathbb{D}_0$ and $\mathbb{D}_1$ monoidal, compatibly

⋆ For $\mathcal{V}\text{-}\mathbb{M}\mathrm{at}$, $(X \otimes Y) = X \times Y$ & $(S \otimes T)((x,y),(z,w)) = S(x,z) \otimes T(y,w)$

■ *Locally closed* monoidal double cats: $\mathbb{D}_0$ and $\mathbb{D}_1$ closed, compatibly

⋆ For $\mathcal{V}\text{-}\mathbb{M}\mathrm{at}$, $[X,Y] = Y^X$ & $H(S,T)$ as before (not ad-hoc anymore!)

■ *Locally presentable* double cats: $\mathbb{D}_0$ and $\mathbb{D}_1$ loc pres, compatibly

Sweedler theory for double categories

We have now transferred all necessary structure to the very top!

Suppose $\mathbb{D}$ is a locally closed symmetric monoidal double category, fibrant and locally presentable. Then there is a parameterized adjunction

$$H(\text{-},\text{-})\colon \mathsf{Cmd}(\mathbb{D})^{\mathrm{op}} \times \mathsf{Mnd}(\mathbb{D}) \to \mathsf{Mnd}(\mathbb{D}) \quad \text{given - ‘convolution’}$$

$$S(\text{-},\text{-})\colon \mathsf{Mnd}(\mathbb{D})^{\mathrm{op}} \times \mathsf{Mnd}(\mathbb{D}) \to \mathsf{Cmd}(\mathbb{D}) \quad \text{new - universal measuring}$$

Moreover, $\mathsf{Mnd}(\mathbb{D})$ is enriched in $\mathsf{Cmd}(\mathbb{D})$.

Not an easy result, but applicable in a straightforward way.

⋆ In $\mathcal{V}$-$\mathbb{M}$at, obtain ($\mathcal{V}$-Cocat)-enrichment of $\mathcal{V}$-Cat, without getting into details of the specific structures as before...

⋆ A $\mathbb{D}$ with single object & vertical 1-cell ‘is’ a monoidal category... back to (co)monoids in monoidal categories❢

Enriched duality in double categories

Suppose $\mathbb{D}$ is a locally closed symmetric monoidal double category, fibrant and locally presentable. The fibration on the left is enriched in the monoidal opfibration on the right

$$\begin{array}{ccc} \mathsf{Mod}(\mathbb{D}) & \qquad & \mathsf{Comod}(\mathbb{D}) \\ \downarrow & & \downarrow \\ \mathsf{Mnd}(\mathbb{D}) & & \mathsf{Cmd}(\mathbb{D}) \end{array}$$

Further directions

So far: established very broad framework for Sweedler theory – enrich monads in comonads, modules in comodules – in general double $\mathbb{D}$.

Next: employ or *extend* theory for interesting examples in other contexts!

- $\mathbb{D} = \mathcal{V}\text{-}\mathbb{M}\mathrm{at}$ gives universal measuring cocategories
- $\mathbb{D} =$ other nicely behaved monoidal double categories?
- $\mathbb{D} = \mathcal{V}\text{-}\mathbb{S}\mathrm{ym}$ *should* give universal measuring *cooperads*...

$\mathbb{D}$	$\mathcal{V}\text{-}\mathbb{M}\mathrm{at}$	$\mathcal{V}\text{-}\mathbb{S}\mathrm{ym}$
objects	sets $X, Y, \ldots$	sets $X, Y, \ldots$
vertical 1-cells	functions $f, g, \ldots$	functions $f, g, \ldots$
horizontal 1-cells $X \to\!\!\!\!\!\!\;\mapsto Y$	matrices $\{S_{x,y}\}$ i.e. $X \times Y \xrightarrow{S} \mathcal{V}$	symmetries $\Sigma(X) \times Y \xrightarrow{S} \mathcal{V}$
2-cells	$S_{x,y} \to T_{fx,gy}$	$S_{x_1,\ldots,x_n;y} \to T_{fx_1,\ldots,fx_n;gy}$

Problem: $\mathcal{V}$-$\mathbb{S}$ym is a double category (✓) which is fibrant (✓) but NOT monoidal (✗)! Before even locally closed (✎) or locally presentable (✎).

> $\mathcal{V}$-$\mathbb{S}$ym is (a special case of) a horizontal Kleisli double category for a horizontal double monad on a fibrant double category.

Gives first two ✓. For ✗, replace with new structure...

> $\mathcal{V}$-$\mathbb{S}$ym admits a normal oplax monoidal structure, given by the arithmetic product of symmetric sequences.

Task: extend results for *oplax* monoid double categories!
$\star$ Single object and vertical 1-cell now reduces to *duoidal* categories...

Goal: enrichment of (non-colored and colored) operads in cooperads, connect to bar-cobar construction and operadic Koszul duality!

Thank you for your attention!

$$
\begin{tikzcd}
{} \arrow[r] \arrow[d] & {} \arrow[d] \\
{} \arrow[r] & {}
\end{tikzcd}
$$

- *Buckley, Fieremans, Vasilakopoulou, Vercruysse*, "Oplax Hopf algebras", Higher structures, 2021
- *Gambino, Garner, Vasilakopoulou*, "Monoidal Kleisli bicategories and the arithmetic product of symmetric sequences", arXiv:2206.06858
- *Hyland, Lopez Franco, Vasilakopoulou*, "Hopf measuring comonoids and enrichment", Proceedings of the LMS, 2017
- *Hyland, Lopez Franco, Vasilakopoulou*, "Measuring comodules and enrichment", arXiv:1703.10137
- *Lopez Franco, Vasilakopoulou*, "Duoidal categories, measuring comonoids and enrichment", arXiv:2005.01340
- *Vasilakopoulou*, "On enriched fibrations", Cahiers de topologie et géométrie différentielle catégoriques, 2018
- *Vasilakopoulou*, "Enriched duality in double categories: V-categories and V-cocategories", Journal of Pure and Applied Algebra, 2019