Combining Problems on RAC Drawings and Simultaneous Graph Drawings

Evmorfia N. Argyriou[1], Michael A. Bekos[1],
Michael Kaufmann[2], and Antonios Symvonis[1]

[1] School of Applied Mathematical & Physical Sciences,
National Technical University of Athens, Greece
{fargyriou,mikebekos,symvonis}@math.ntua.gr
[2] University of Tübingen, Institute for Informatics, Germany
mk@informatik.uni-tuebingen.de

1 Introduction and Problem Definition

We present an overview of the first combinatorial results for the so-called *geometric RAC simultaneous drawing problem* (or *GRacSim* drawing problem, for short), i.e., a combination of problems on geometric RAC drawings [3] and geometric simultaneous graph drawings [2]. According to this problem, we are given two planar graphs $G_1 = (V, E_1)$ and $G_2 = (V, E_2)$ that share a common vertex set but have disjoint edge sets, i.e., $E_1 \subseteq V \times V$, $E_2 \subseteq V \times V$ and $E_1 \cap E_2 = \emptyset$. The main task is to place the vertices on the plane so that, when the edges are drawn as straight-lines, (i) each graph is drawn planar, (ii) there are no edge overlaps, and, (iii) crossings between edges in E_1 and E_2 occur at right angles.

A closely related problem is the following: *Given a planar embedded graph G, determine a geometric drawing of G and its dual G^* (without the face-vertex corresponding to the external face) such that: (i) G and G^* are drawn planar, (ii) each vertex of the dual is drawn inside its corresponding face of G and, (iii) the primal-dual edge crossings form right-angles.* We refer to this problem as the *geometric Graph-Dual RAC simultaneous drawing problem* (or *GDual-GRacSim* for short).

2 Results

A detailed presentation of our results (including technical proofs) is available as a technical report [1]. The following theorem establishes that if two graphs always admit a geometric simultaneous drawing, it is not necessary that they also admit a GRacSim drawing.

Theorem 1. *There exists a wheel and a cycle which do not admit a* GRacSim *drawing.*

For the case of a path $\mathcal{P}$ and a matching $\mathcal{M}$, we can prove that a GRacSim drawing always exists. The basic idea of our algorithm is to identify in the graph induced by the union of $\mathcal{P}$ and $\mathcal{M}$ a set of cycles $\mathcal{C}_1, \ldots, \mathcal{C}_k$, $k \leq n/4$, such that:

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(i) $|E(\mathcal{C}_1)| + \ldots + |E(\mathcal{C}_k)| = n$, (ii) $\mathcal{M} \subseteq \mathcal{C}_1 \cup \ldots \cup \mathcal{C}_k$, and, (iii) the edges of cycle $\mathcal{C}_i$, $i = 1, \ldots, k$ alternate between edges of $\mathcal{P}$ and $\mathcal{M}$. The edges of the cycle collection do not cross each other, while the remaining ones introduce right-angle crossings. The following theorem summarizes our result.

Theorem 2. *A path and a matching always admit a* GRacSim *drawing on an* $(n/2+1) \times n/2$ *integer grid. The drawing can be computed in linear time.*

We can extend the algorithm that produces a GRacSim drawing of a path and a matching to also cover the case of a cycle $\mathcal{C}$ and a matching $\mathcal{M}$. The idea is simple. If we remove an edge from the input cycle, the remaining graph is a path $\mathcal{P}$. So, we can apply the developed algorithm and obtain a GRacSim drawing of $\mathcal{P}$ and $\mathcal{M}$, in which the insertion of the edge that closes the cycle can be done without introducing any crossings by augmenting the total area of the drawing.

Theorem 3. *A cycle and a matching always admit a* GRacSim *drawing on an* $(n+2) \times (n+2)$ *integer grid. The drawing can be computed in linear time.*

Corollary 1. *Let G be a simple connected graph that can be decomposed into a matching and either a hamiltonian path or a hamiltonian cycle. Then, G is a RAC graph.*

For the GDual-GRacSim drawing problem, we can show by an example that it is not always possible to compute a GDual-GRacSim drawing if the input graph is an arbitrary planar graph. This is summarized in the following theorem.

Theorem 4. *Given a planar embedded graph G, a* GDual-GRacSim *drawing of G and its dual G^* does not always exist.*

For the more restricted case of outerplanar graphs, we can state the following theorem, which is based on a recursive geometric construction that computes a GDual-GRacSim drawing of G and its dual.

Theorem 5. *Given an outerplane embedding of an outerplanar graph G, it is always feasible to determine a* GDual-GRacSim *drawing of G and its dual G^*.*

Our study raises several open problems. It would be interesting to identify other non-trivial classes of graphs, besides a matching and either a path or a cycle, that admit a GRacSim drawing. For the classes where GRacSim drawings are not possible, study drawings with bends. Study the required drawing area.

References

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