COMPUTATIONAL FINANCE

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Exercise 2

Set-up. Consider the Heston stochastic volatility model (under a risk neutral measure), provided by the SDEs

$$\begin{aligned} \mathrm{d}X_t &= \big(r - \frac{1}{2}V_t\big)\mathrm{d}t + \sqrt{V_t}\mathrm{d}W_t, \quad X_0 = x, \\ \mathrm{d}V_t &= \kappa(\theta - V_t)\mathrm{d}t + \eta\sqrt{V_t}\mathrm{d}\overline{W}_t, \quad V_0 = v, \end{aligned} \tag{1}$$

where X denotes the logarithm of the asset price and V the variance process. The parameters satisfy $r \in \mathbb{R}$, $\kappa, \theta, \eta \in \mathbb{R}_+$, the initial values are $x \in \mathbb{R}, v \in \mathbb{R}_+$, and the Brownian motions $W, \overline{W}$ are correlated with parameter $\rho \in [-1, 1]$. An Euler discretization of this SDE was provided in the class.

Tasks.

(1) Compute the prices of a call option and the 95% confidence intervals with the Euler Monte Carlo method.
(2) Study empirically the convergence of the Euler scheme.

Data.

- Spot price $S_0 = 100$, interest rate $r = 0\%$.
- Maturity $T = 5$, strike prices $K = \{80, 100, 120\}$.
- Heston parameters: $\kappa = 1$, $\theta = v = 9\%$, $\eta = 1$, $\rho = -0.3$.

Submit.

- The source code (in `scilab`/`matlab`/`C`/`python`/...). The source code should include sufficient documentation.
- A PDF file explaining how the code was developed and discussing the results (preferably written in LaTeX).
- Submit everything per e-mail to

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  in a zip file named: `Exercise_2_Surname_Name`.
- Deadline: **July 3, 2016**.

Hints.

- For the Heston model, there is no closed-form expression for the option prices. Thus, you will first have to compute a sufficiently precise reference price using the Euler Monte Carlo method with N_{ref} (number of steps) and M_{ref} (number of trajectories) (very) large.[1]
- Alternatively, you can compute the value using the online calculator available by INRIA `https://quanto.inria.fr/premia/koPremia`, by selecting "equity_stochastic_volatility" and then "Heston1dim".

[1] You will have to determine what this means. It may well be that your subsequent error analysis will show that the original choice of N_{ref} or M_{ref} was not big enough. Then you will have to restart the experiments with new choices for N_{ref} and/or M_{ref}.

- Choose a sequence of N_i and corresponding M_i and consider the corresponding weak errors, i.e., the (absolute) differences of the option prices obtained with (N_i, M_i) and the reference price obtained previously. Compute the "empirical rate of convergence", i.e., determine the parameters C and (above all) α in a regression $\text{error}_i \sim C N_i^{-\alpha}$.
- As we want to study the convergence of the discretization error, we need to choose M_i such that the Monte Carlo error is significantly smaller than the discretization error. In other words, the confidence interval around error_i should be considerably smaller than error_i itself.[2]
- Plot your observed errors error_i together with the confidence intervals around them and with the reference curve $C N_i^{-\alpha}$. As usual, you should use a log-log-plot. You might use Figure 3.5 in the lecture notes as an example of how your plot could look like. The plot gives you a visual check of whether the M_i have been sufficiently large. (Figure 3.1 is an extreme case, though. You should not try to make the confidence interval comparably small in the homework!)
- The submitted pdf-file should contain the plot as well as the underlying data and the estimated parameters C and α.
- Do not throw away your code and, in particular, your reference price. You will need them for the third homework, as well.

[2] This means that you can practically rule out that the Monte Carlo error has a significant effect on the estimated α.